4.5.2 Coulomb's shear strength criterion

某一岩層，強度受莫耳－庫倫((**Mohr－Coulomb**)破壞準則控制，其凝聚力**C=20MPa**，內摩擦角ϕ=**40º**，今受到最大主應力爲**100Mpa**，最小主應力爲**15MPa**的應力作用。

(三)破壞時，斷裂面上的有效正應力與剪應力分別是多少？

$$\alpha = 45 + \frac{\phi}{2} = 65$$

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} - \frac{\sigma_1 - \sigma_3}{2}\cos(\pi - 2\times 65) = 15MPa$$

$$\tau = \frac{\sigma_1 - \sigma_3}{2}\sin(\pi - 2\times 65) = 32.56MPa$$

4.5.3 Griffith crack theory

Griffith (1921) postulated that fracture of brittle materials, such as steel and glass, is initiated at tensile stress concentrations at the tips of minute, thin cracks (now referred to as Griffith cracks) distributed throughout an otherwise isotropic, elastic material. Griffith based his determination of the conditions under which a crack would extend off his energy instability concept:

$$\frac{\partial}{\partial c}(W_d - W_e) \leq 0$$

where c is a crack length parameter, W_e is the elastic strain energy stored around the crack and W_d is the surface energy of the crack surfaces.

4.5.3 Griffith crack theory

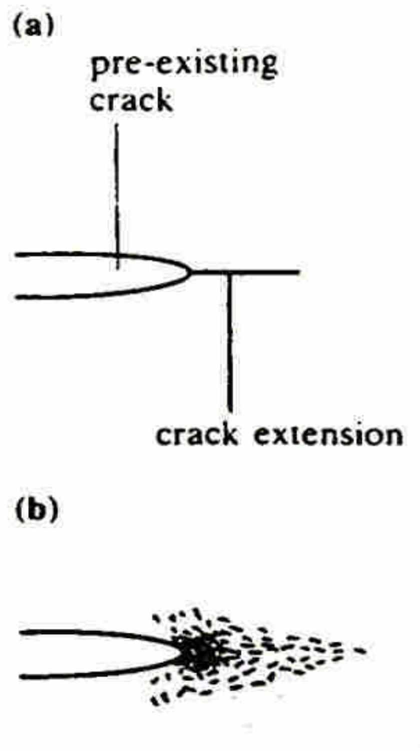


Figure 4.25 Extension of a pre-existing crack. (a) Griffith's hypothesis. (b) the actual case for rock.

$$\sigma \geq \sqrt{\frac{2E\alpha}{\pi c}} \qquad \textbf{(4.19)}$$

- It is important to note that it is the surface energy, α , which is the fundamental material property involved here. In this case, it is preferable to treat α as an apparent surface energy to distinguish it from the true surface energy which may have a significantly smaller value.

4.5.3 Griffith crack theory

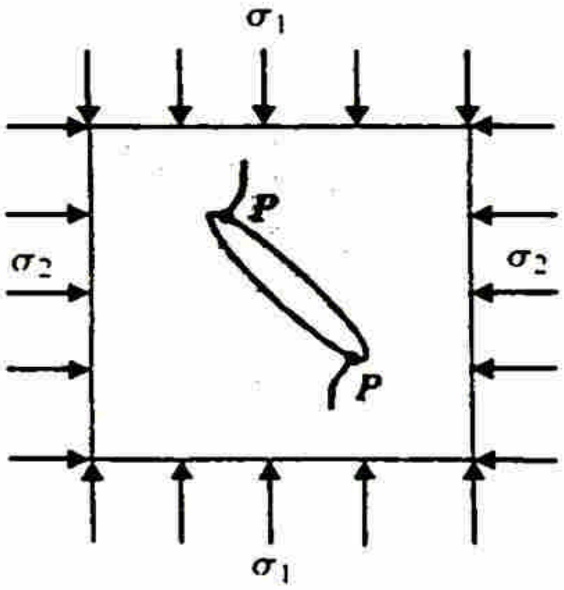
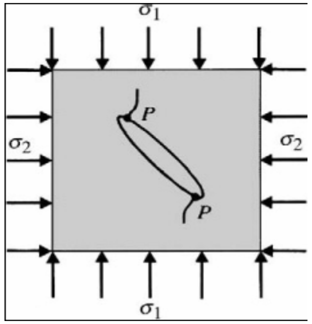


Figure 4.26 Griffith crack model for plane compression.

- Griffith (1924) extended his theory to the case of applied compressive stresses. Neglecting the influence of friction on the cracks which will close under compression, and assuming that the elliptical crack will propagate from the points of maximum tensile stress concentration (P in Figure 4.26).



4.5.3 Griffith crack theory

Griffith obtained the following criterion for crack extension in plane compression:

$$(\sigma_1 - \sigma_2)^2 - 8T_0(\sigma_1 + \sigma_2) = 0 \quad if \;\; \sigma_1 + 3\sigma_2 > 0$$
$$\sigma_2 + T_0 = 0 \quad if \;\; \sigma_1 + 3\sigma_2 < 0 \qquad \textbf{(4.20)}$$

where T_0 is the uniaxial tensile strength of the uncracked material (a positive number).

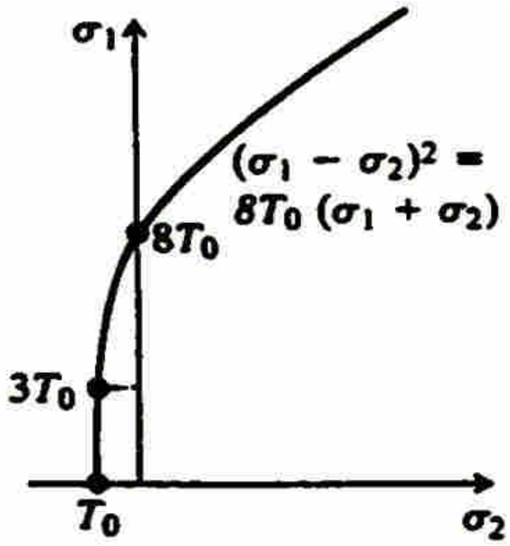
This criterion can also be expressed in terms of the shear stress, τ , and the normal stress, σ_n, acting on the plane containing the major axis of the crack:

$$\tau^2 = 4T_0(\sigma_n + T_0) \qquad \textbf{(4.21)}$$

Griffith crack theory

$$(\sigma_1 - \sigma_2)^2 - 8T_0(\sigma_1 + \sigma_2) = 0 \quad if \;\; \sigma_1 + 3\sigma_2 > 0 \quad \textbf{(6)}$$

$$\sigma_2 + T_0 = 0 \quad if \;\; \sigma_1 + 3\sigma_2 < 0$$



where T_0 is the uniaxial tensile strength of the uncracked material (a positive number).

Griffith crack theory

The function of Mohr circle is :

$$\tau^2 + (\sigma_n - \frac{\sigma_1 + \sigma_2}{2})^2 = (\frac{\sigma_1 - \sigma_2}{2})^2 \qquad \textbf{(7)}$$

$$\sigma_m = \frac{\sigma_1 + \sigma_2}{2} \quad , \quad \tau_m = \frac{\sigma_1 - \sigma_2}{2} \qquad \textbf{(8)}$$

Substituting for Eq.(8) in Eq.(6) gives

$$\tau_m{}^2 = 4T_0 \times \sigma_m \qquad \textbf{(9)}$$

Griffith crack theory

$$f = \tau^2 + (\sigma_n - \sigma_m)^2 - 4T_0 \times \sigma_m \quad \textbf{(10)}$$

$$\frac{\partial f}{\partial \sigma_m} = 0 \quad \text{gives} \quad \sigma_m = \sigma_n + 2T_0 \quad \textbf{(11)}$$

Substituting for Eq.(11) in Eq.(7) gives

$$\tau^2 + 4T_0^2 = 4T_0 \times (\sigma_n + 2T_0) \quad \textbf{(12)}$$

Eq.(12) reduces to

(13)

$$\tau^2 = 4T_0 \times (\sigma_n + T_0)$$

4.5.3 Griffith crack theory

The envelopes given by equations 4.20 and 4.21 are shown in Figure 4.27.

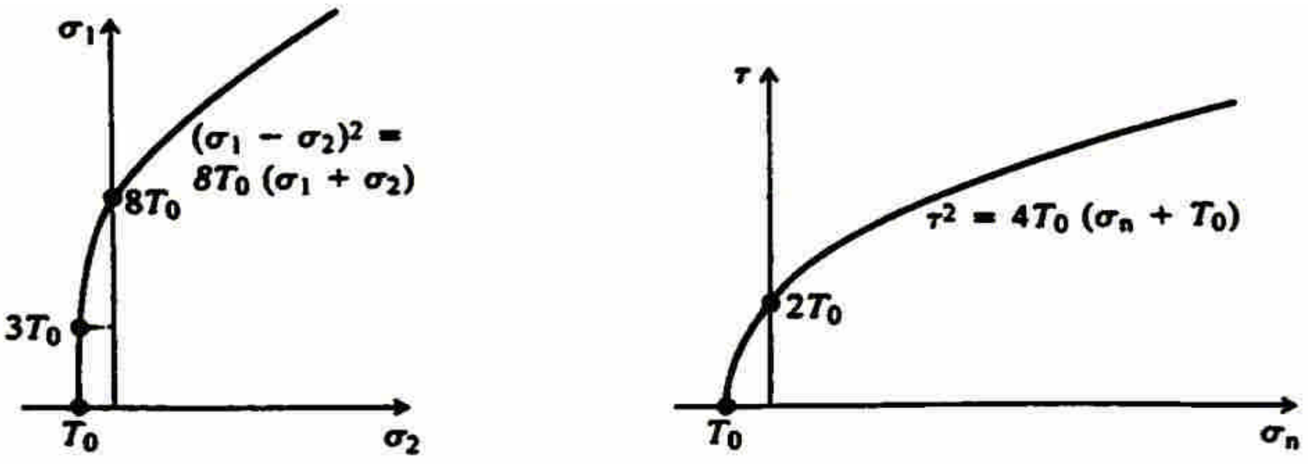


Figure 4.27 Griffith envelopes for crack extension in plane compression.

4.5.3 Griffith crack theory

Accordingly, a number of modifications to Griffith's solution were introduced (see Paterson, 1978 and Jaeger and Cook, 1979 for details).

These criteria do not find practical use today.

However, Griffith's energy instability concept has formed the basis of the new science of fracture mechanics which is being applied increasingly to the study of the fracture of rock.

4.5.3 Griffith crack theory

- The use of this approach with an apparent surface energy taken as the basic material property has been able to explain many observations of apparent size effects and to reconcile the results of different types of indirect tension test on rock.

4.5.4 Empirical criteria

Bieniawski (1974) found that the peak triaxial strengths of a range of rock types were well represented by the criterion

$$\frac{\sigma_1}{\sigma_c} = 1 + A\left(\frac{\sigma_3}{\sigma_c}\right)^k \quad \textbf{(4.23)}$$

or

$$\frac{\tau_m}{\sigma_c} = 0.1 + B\left(\frac{\sigma_m}{\sigma_c}\right)^c \quad \textbf{(4.24)}$$

where $\tau_m = \frac{1}{2}(\sigma_1 - \sigma_3)$ and $\sigma_m = \frac{1}{2}(\sigma_1 + \sigma_3)$.

$$\frac{\sigma_1}{\sigma_c} = 1 + A\left(\frac{\sigma_3}{\sigma_c}\right)^k \qquad \frac{\tau_m}{\sigma_c} = 0.1 + B\left(\frac{\sigma_m}{\sigma_c}\right)^c$$

4.5.4 Empirical criteria

Bieniawski found that, for the range of rock types tested, $k \cong 0.75$ and $c \cong 0.90$.

Both A and B take relatively narrow ranges for the rock types tested. (Two parameters model)

Table 4.1 Constants in Bieniawski's empirical strength criterion (after Bieniawski, 1974).

Rock type	*A*	B
norite	5.0	0.8
quartzite	4.5	0.78
sandstone	4.0	0.75
siltstone	3.0	0.70
mudstone	3.0	0.70

4.5.4 Empirical criteria

Hoek and Brown (1980):

$$\frac{\sigma_1}{\sigma_c} = \frac{\sigma_3}{\sigma_c} + (m\frac{\sigma_3}{\sigma_c} + s)^{1/2} \qquad \textbf{(4.25)}$$

where m varies with rock type. ***S=1.0*** for intact rock. Analysis of published strength data suggests that m increases with rock type in the following general way:

(a) $m \cong 7$ for carbonate rocks with well developed crystal cleavage (dolomite,limestone, marble);

(b) $m \cong 10$ for lithofied argillaceous rocks (mudstone, siitstone, shale, slate);

$$\frac{\sigma_1}{\sigma_c} = \frac{\sigma_3}{\sigma_c} + \left(m\frac{\sigma_3}{\sigma_c} + 1.0\right)^{1/2}$$

4.5.4 Empirical criteria

(c) $m \cong 15$ for arenaceous rocks(砂質岩) with strong crystals and poorly developed crystal cleavage (sandstone, quartzite);

(d) $m \cong 17$ for fine-grained polyminerallic igneous crystalline rocks (andesite, dolerite, diabase, rhyolite);

(e) $m \cong 25$ for coarse-grained polymineralic igneous and metamorphic rocks (amphibolite, gabbro, gneiss, granite, norite, quartz-diorite).

4.5.4 Empirical criteria

$$\frac{\sigma_1}{\sigma_c} = \frac{\sigma_3}{\sigma_c} + (m\frac{\sigma_3}{\sigma_c} + 1.0)^{1/2}$$

Normalised peak strength envelopes for sandstones

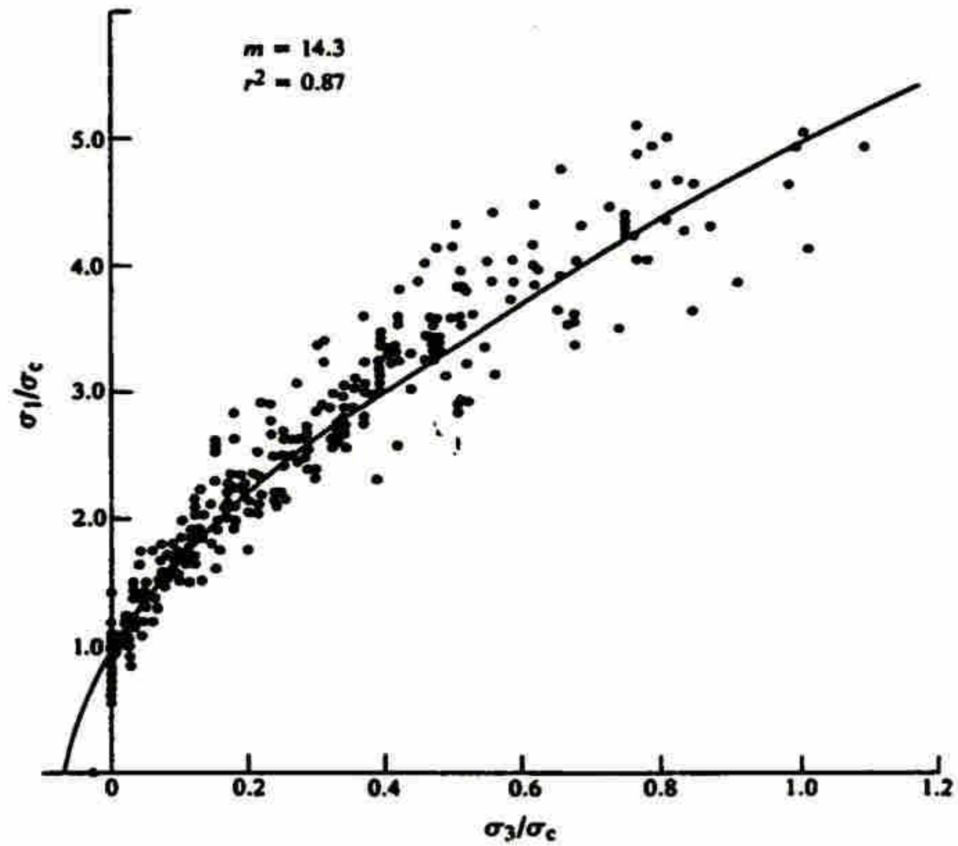


Figure 4.28 Normalised peak strength envelope for sandstones (after Hoek and Brown, 1980).

4.5.4 Empirical criteria

$$\frac{\sigma_1}{\sigma_c} = \frac{\sigma_3}{\sigma_c} + \left(m\frac{\sigma_3}{\sigma_c} + 1.0\right)^{1/2}$$

Normalised peak strength envelopes for granites

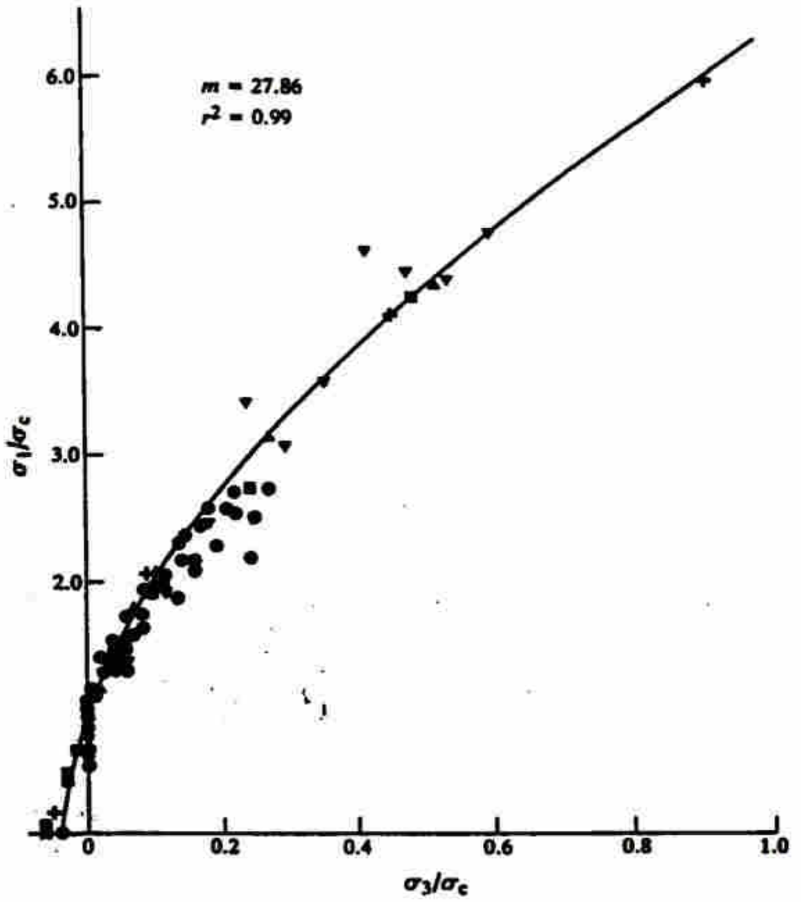


Figure 4.29 Normalised peak strength envelope for granites (after Hoek and Brown, 1980).

Generalised Hoek-Brown criterion

For jointed rock masses

$$\frac{\sigma_1}{\sigma_c} = \frac{\sigma_3}{\sigma_c} + \left(m\frac{\sigma_3}{\sigma_c} + 1.0\right)^{1/2}$$

$$\sigma_1' = \sigma_3' + \sigma_{ci}\left(m_b\frac{\sigma_3'}{\sigma_{ci}} + s\right)^a \qquad \textbf{(1)}$$

where σ_1' and σ_3' are the maximum and minimum effective stress at failure,

m_b is the value of the Hoek-Brown constant for the rock mass,

s and a are constants which depend upon the rock mass characteristics

σ_{ci} is the uniaxial compressive strength of the intact rock

Generalised Hoek-Brown criterion

For the intact rock

$$\sigma_1' = \sigma_3' + \sigma_{ci}\left(m_b \frac{\sigma_3'}{\sigma_{ci}} + s\right)^a$$

For the intact rock pieces that make up the rock mass Eq.(1) simplifies to :

$$\sigma_1' = \sigma_3' + \sigma_{ci}\left(m_i \frac{\sigma_3'}{\sigma_{ci}} + 1\right)^{0.5} \qquad \textbf{(2)}$$

where σ_{ci} **is the uniaxial compressive strength of the intact rock pieces**

m_i **is the value of the Hoek-Brown constant for the intact rock**

Generalised Hoek-Brown criterion
Geological Strength Index

$$\sigma_1' = \sigma_3' + \sigma_{ci}\left(m_b \frac{\sigma_3'}{\sigma_{ci}} + s\right)^a$$

- The Geological Strength Index (GSI)(Hoek,1995)

$$m_b = m_i \exp\left(\frac{GSI - 100}{28}\right)$$

For GSI > 25

$$s = \exp\left(\frac{GSI - 100}{9}\right) \quad , \quad a = 0.5$$

For GSI < 25

$$s = 0 \quad , \quad a = 0.65 - \frac{GSI}{200} \qquad \textbf{(3)}$$

Table 8.4: Estimation of constants m_b / m_i, s, a, deformation modulus E and the Poisson's ratio ν for the Generalised Hoek-Brown failure criterion based upon rock mass structure and discontinuity surface conditions. Note that the values given in this table are for an *undisturbed* rock mass.

GENERALISED HOEK-BROWN CRITERION

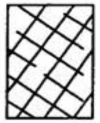
$$\sigma_1' = \sigma_3' + \sigma_c \left(m_b \frac{\sigma_3'}{\sigma_c} + s \right)^a$$

σ_1' = major principal effective stress at failure

σ_3' = minor principal effective stress at failure

σ_c = uniaxial compressive strength of *intact* pieces of rock

m_b, s and a are constants which depend on the composition, structure and surface conditions of the rock mass

STRUCTURE	SURFACE CONDITION	VERY GOOD Very rough, unweathered surfaces	GOOD Rough, slightly weathered, iron stained surfaces	FAIR Smooth, moderately weathered or altered surfaces	POOR Slickensided, highly weathered surfaces with compact coatings or fillings containing angular rock fragments	VERY POOR Slickensided, highly weathered surfaces with soft clay coatings or fillings
 BLOCKY -very well interlocked undisturbed rock mass consisting of cubical blocks formed by three orthogonal discontinuity sets	m_b/m_i s a E_m ν *GSI*	0.60 0.190 0.5 75,000 0.2 85	0.40 0.062 0.5 40,000 0.2 75	0.26 0.015 0.5 20,000 0.25 62	0.16 0.003 0.5 9,000 0.25 48	0.08 0.0004 0.5 3,000 0.25 34
 VERY BLOCKY-interlocked, partially disturbed rock mass with multifaceted angular blocks formed by four or more discontinuity sets	m_b/m_i s a E_m ν *GSI*	0.40 0.062 0.5 40,000 0.2 75	0.29 0.021 0.5 24,000 0.25 65	0.16 0.003 0.5 9,000 0.25 48	0.11 0.001 0.5 5,000 0.25 38	0.07 0 0.53 2,500 0.3 25
 BLOCKY/SEAMY-folded and faulted with many intersecting discontinuities forming angular blocks	m_b/m_i s a E_m ν *GSI*	0.24 0.012 0.5 18,000 0.25 60	0.17 0.004 0.5 10,000 0.25 50	0.12 0.001 0.5 6,000 0.25 40	0.08 0 0.5 3,000 0.3 30	0.06 0 0.55 2,000 0.3 20
 CRUSHED-poorly interlocked, heavily broken rock mass with a mixture of angular and rounded blocks	m_b/m_i s a E_m ν *GSI*	0.17 0.004 0.5 10,000 0.25 50	0.12 0.001 0.5 6,000 0.25 40	0.08 0 0.5 3,000 0.3 30	0.06 0 0.55 2,000 0.3 20	0.04 0 0.60 1,000 0.3 10

Note 1: The in situ deformation modulus E_m is calculated from Equation 4.7 (page 47, Chapter 4). Units of E_m are MPa.

4.5.5 Yield criteria based on plasticity theory

The total strain increment $\left\{\dot{\varepsilon}\right\}$ is the sum of the elastic and plastic strain increments

$$\left\{\dot{\varepsilon}\right\} = \left\{\dot{\varepsilon}^{e}\right\} + \left\{\dot{\varepsilon}^{p}\right\} \qquad \textbf{(4.25)}$$

A plastic potential function, $Q(\{\sigma\})$, is defined such that

$$\left\{\dot{\varepsilon}^{p}\right\} = \lambda \left\{\frac{\partial Q}{\partial \sigma}\right\} \qquad \textbf{(4.26)}$$

where λ is a non-negative constant of proportionality which may vary throughout the loading history.

4.5.5 Yield criteria based on plasticity theory

Thus, from the incremental form of equation 2.38 and equations 4.25 and 4.26

$$\left\{\dot{\varepsilon}\right\} = [D]^{-1}\left\{\dot{\sigma}\right\} + \lambda\left\{\frac{\partial Q}{\partial \sigma}\right\} \qquad \textbf{(4.27)}$$

where $[D]$ is the elasticity matrix.

It is also necessary to be able to define the stress states at which yield will occur and plastic deformation will be initiated. For this purpose, a yield function, $F(\{\sigma\})$, is defined such that $F = 0$ at yield. If $Q = F$, the flow law is said to be associated.

4.5.5 Yield criteria based on plasticity theory

In this case, the vectors of $\{\sigma\}$ and $\{\dot{\varepsilon}^{p}\}$ are orthogonal as illustrated in Figure 4.30. This is known as the normality condition.

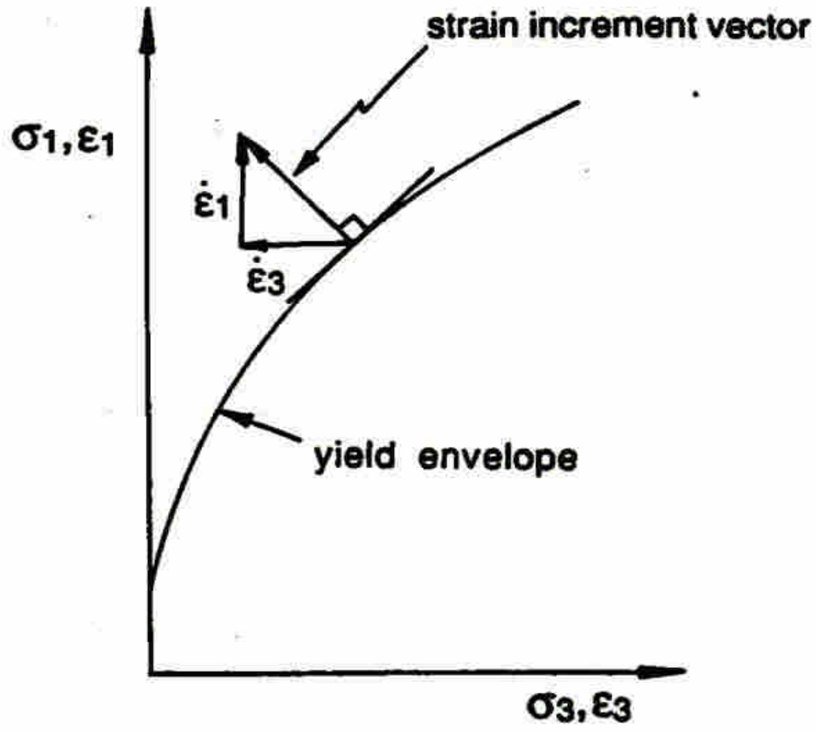


Figure 4.30 The normality condition of the associated flow rule.

4.5.5 Yield criteria based on plasticity theory

For isotropic hardening and associated flow, elastoplastic stress and increments may be related by

the equation

$$\left\{\dot{\sigma}\right\} = \left[D^{ep}\right]\left\{\dot{\varepsilon}\right\}$$

where

$$\left[D^{ep}\right] = [D] - \frac{[D]\left\{\frac{\partial Q}{\partial \sigma}\right\}\left\{\frac{\partial F}{\partial \sigma}\right\}^{T}[D]}{A + \left\{\frac{\partial F}{\partial \sigma}\right\}^{T}[D]\left\{\frac{\partial Q}{\partial \sigma}\right\}}$$

In which

$$A = -\frac{1}{\lambda}\frac{\partial F}{\partial K}dK$$

where K is a hardening parameter such that yielding occurs when

$$dF = \left\{\frac{\partial F}{\partial \sigma}\right\}^T \left\{\dot{\sigma}\right\} + \frac{\partial F}{\partial K} = 0$$

4.5.5 Yield criteria based on plasticity theory

The concepts of associated plastic flow were developed for perfectly plastic and strain-hardening metals using yield functions such as those of Tresca and von Mises which are independent of the hydrostatic component of stress (Hill.1950).

Although these concepts have been found to apply to some geological materials, it cannot be assumed that they will apply to pressure-sensitive materials such as

rocks in which brittle fracture and dilatancy typically occur (Rudnicki and Rice,1975).

4.5.5 Yield criteria based on plasticity theory

These functions are often of the form $F(I_1, J_2) = 0$

where I_1 is the first invariant of the stress tensor and J_2 is the second invariant of the deviator stress tensor (section 2.4), i.e.

$$J_2 = \frac{1}{2}(S_1^2 + S_2^2 + S_3^2)$$

$$= \frac{1}{6}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]$$

4.5.5 Yield criteria based on plasticity theory

More complex functions also include the third invariant of the deviator stress tensor $J_3 = S_1 S_2 S_3$. For example, Desai and Salami (1987) were able to obtain excellent fits to peak strength (assumed synonymous with yield) and stress-strain data for a sandstone, a granite and a dolomite using the yield function

$$F = J_2 - \left(\frac{\alpha}{\alpha_0^{n-2}} I_1^n + I_1^2 \right) \left(1 - \beta \frac{J_3^{1/3}}{J_2^{1/2}} \right)^m$$

where α , β , m and n are material parameters and

α_0 is one unit of stress.