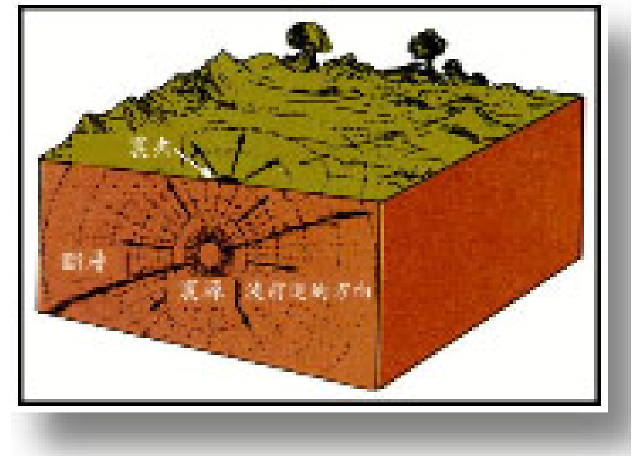
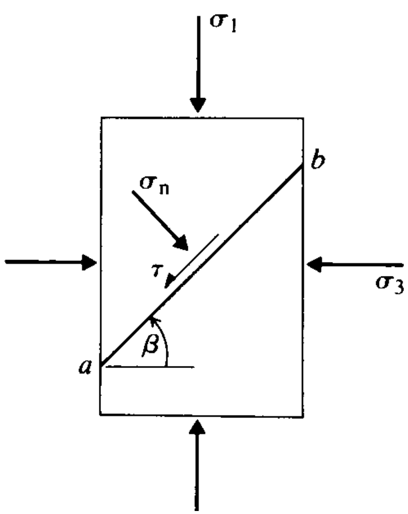
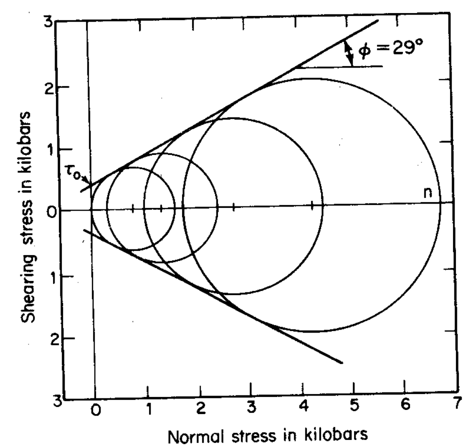
Strength

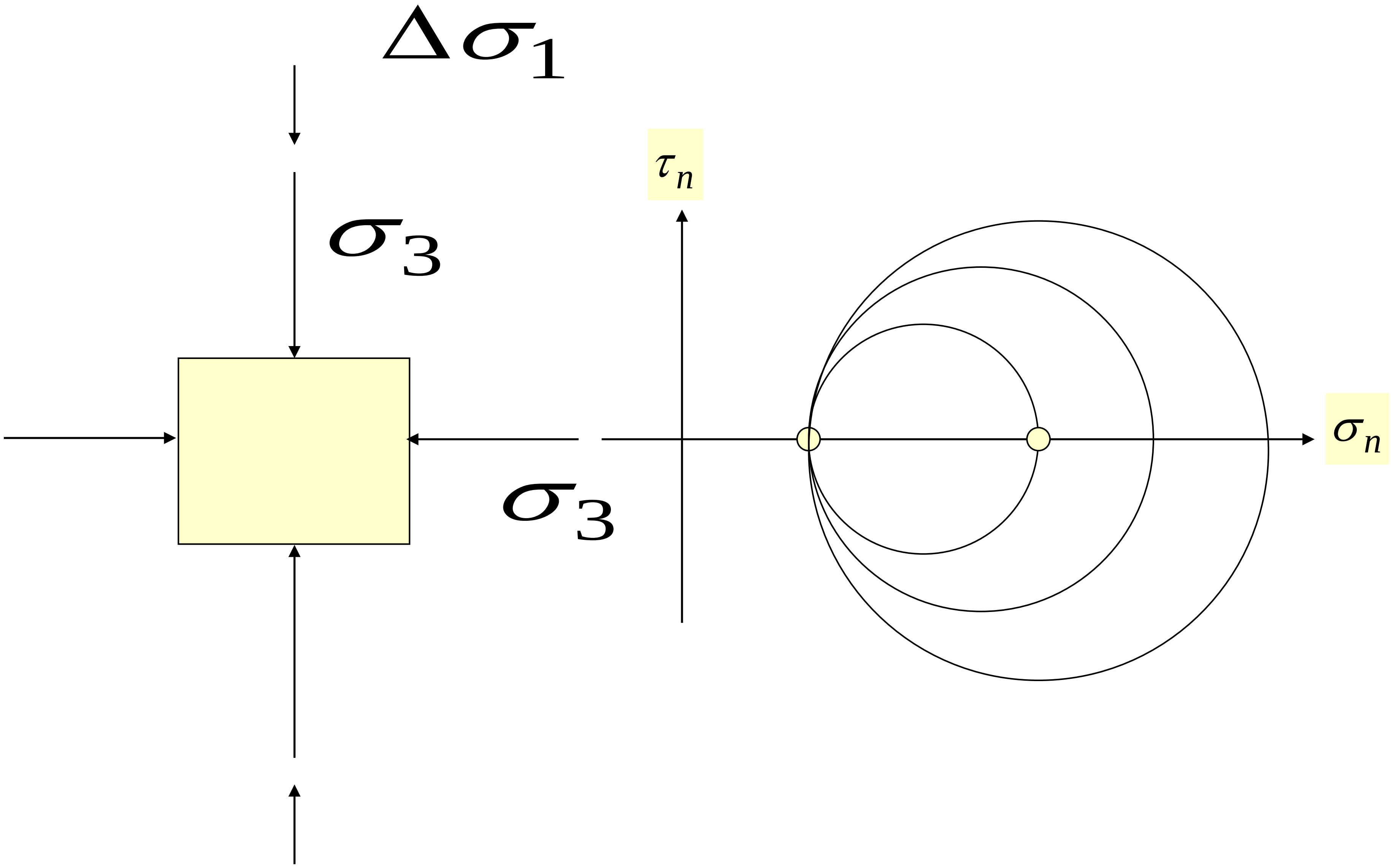


- A stress state that the materials cannot sustain
- Failure





Stress path (stress state change)



Stress path (stress state change)

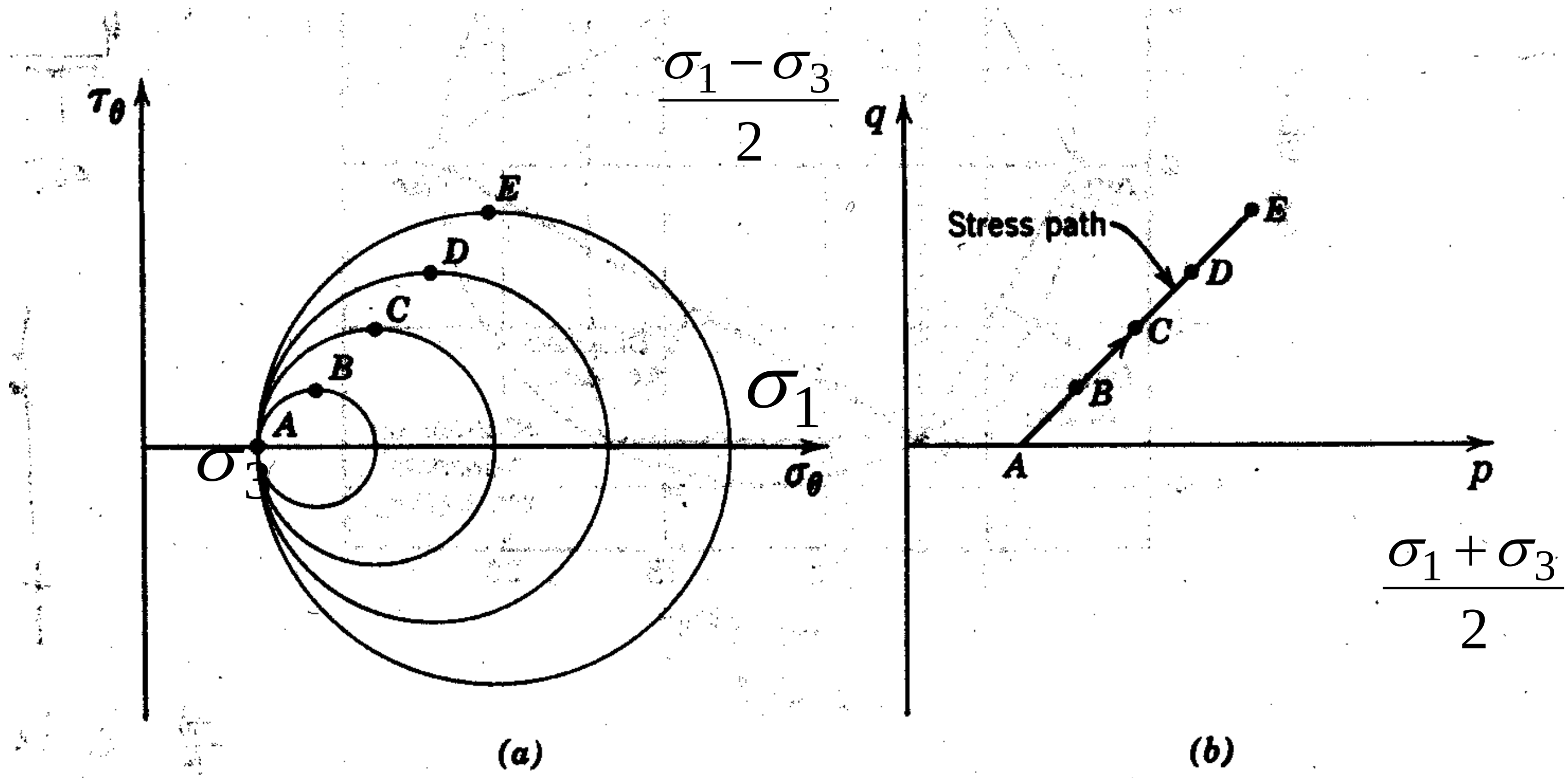
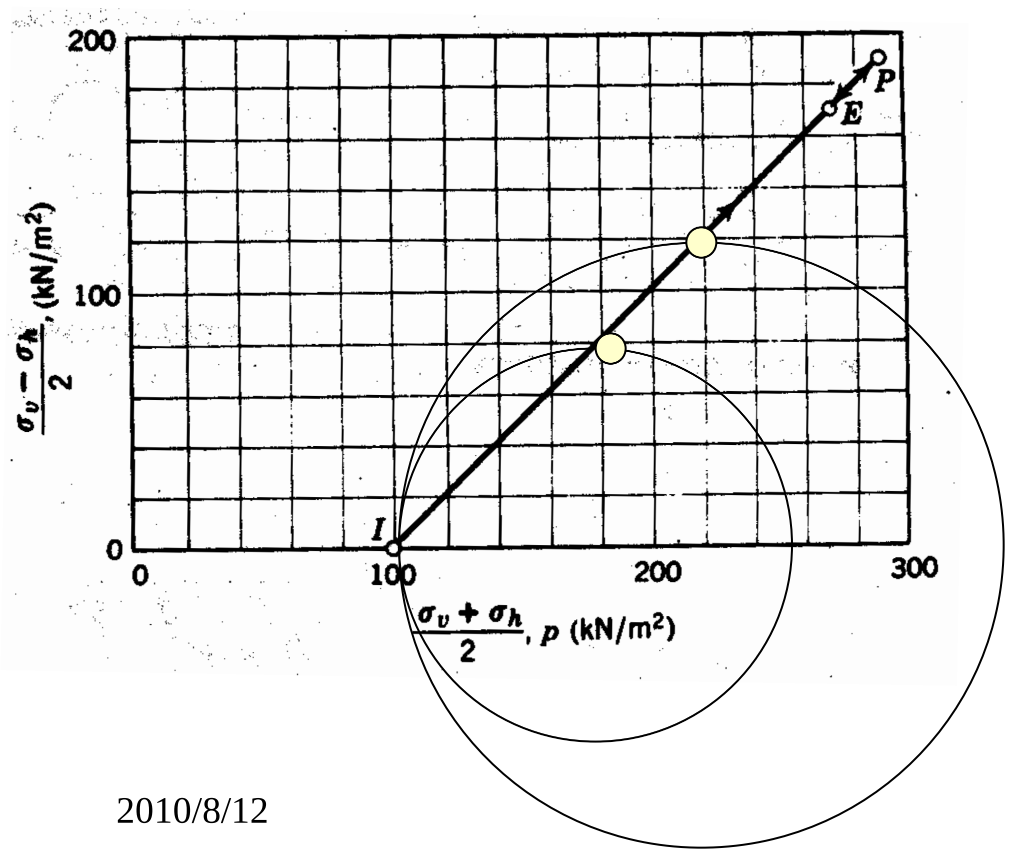
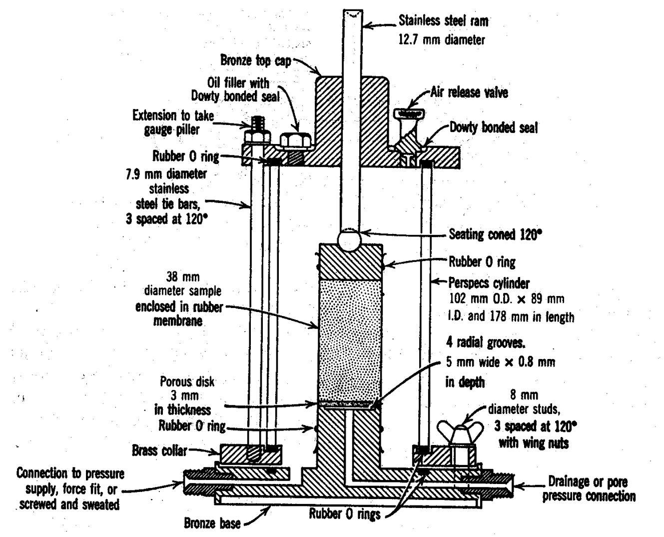
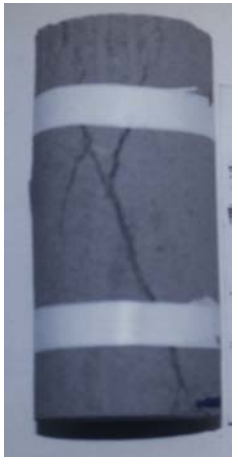


Fig. 8.10 Representation of successive states of stress as σ_1 increases with σ_3 constant. Points *A*, *B*, etc., represent the same stress conditions in both diagrams. (*a*) Mohr circles. (*b*) *p*-*q* diagram.

A stress state that the materials cannot sustain







A stress state that the materials cannot sustain

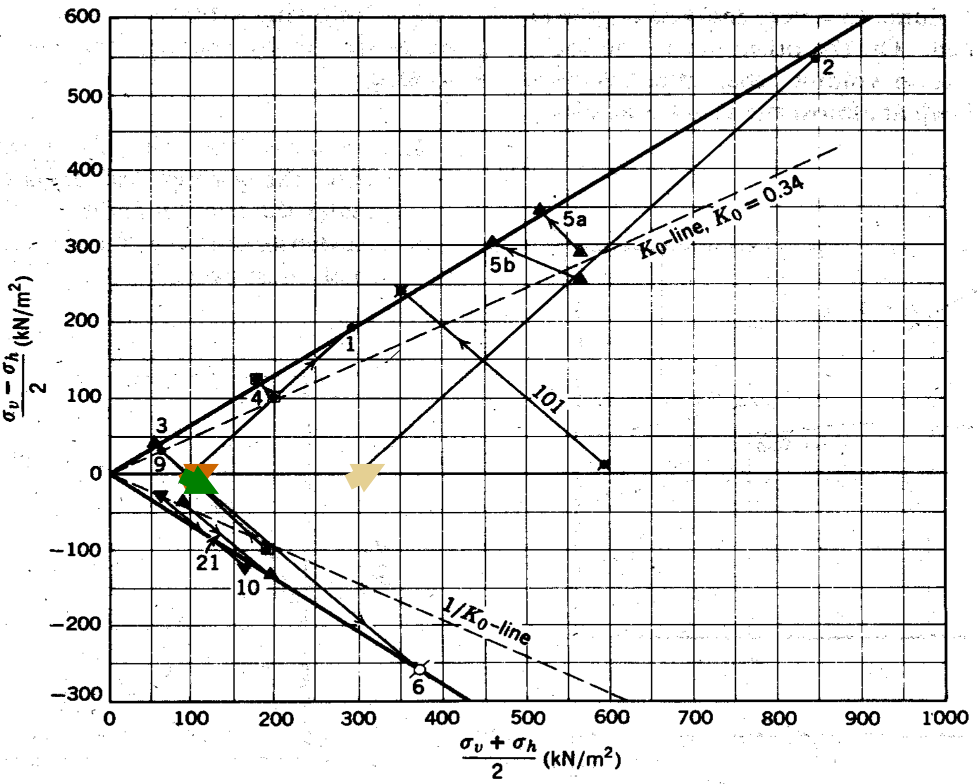
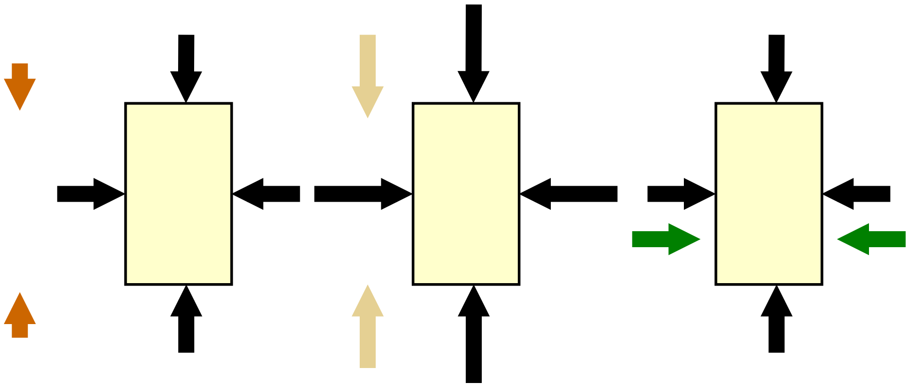
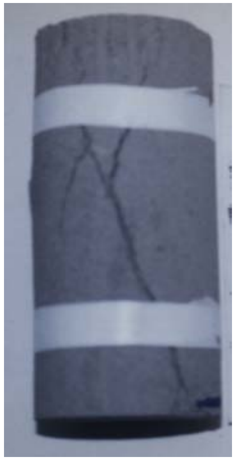


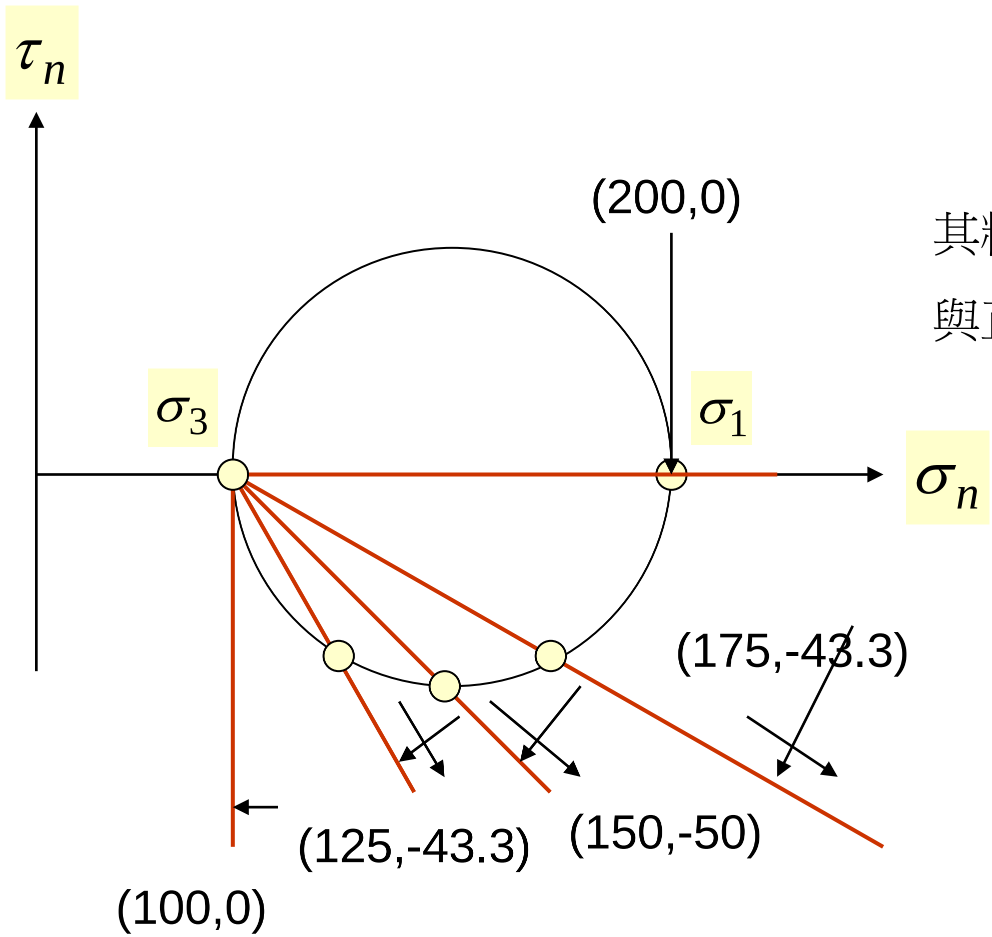
Fig. 10.20 Stress paths for various loadings.

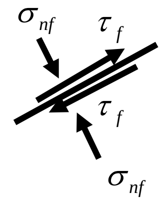




何謂 Mohr-Coulomb failure criterion

Coulomb(1776) 提出剪力強度理論應用於粒狀材料

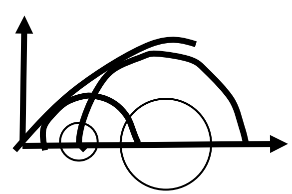




$$\tau_f = \sigma_{nf} \cdot \tan\phi$$

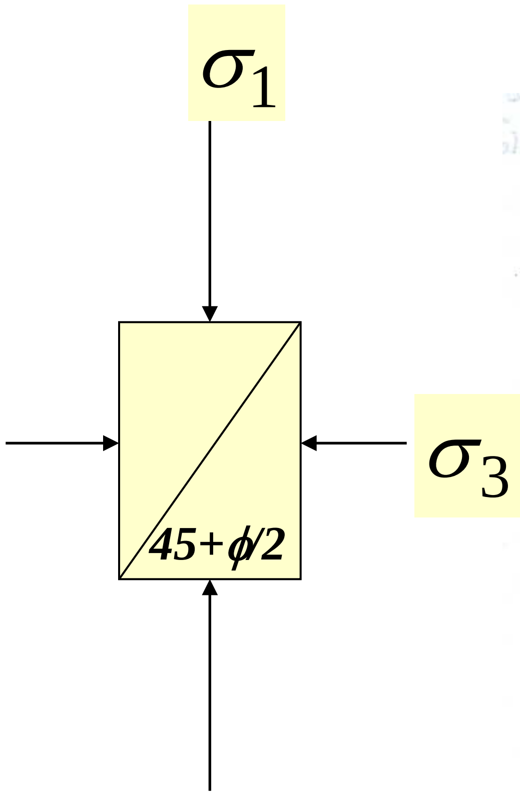
其精神為在某”特定”面上，其剪力強度與與正向力成正比；即該面為 frictional bonds

Mohr (1882) 發現可以圓方程式表示材料應力狀態(Mohr circle)，同時，發現材料破壞時之摩爾圓將可為特定包絡線所包覆



其精神為在某特定 stress state，材料將破壞，實際上並未牽涉到破壞面及破壞面上之剪力或正向力(Mohr envelop實際上為一曲線)

若假設破壞準則符合摩爾庫倫破壞準則(直線破壞包絡線)破壞面是哪一個?與水平面之夾角?



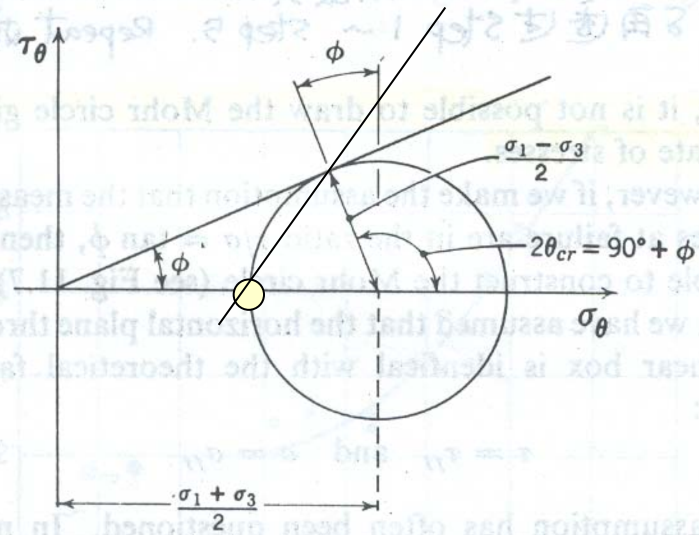


Fig. 11.5 Relations between ϕ and principal stresses at failure.

$$\sin\phi = \frac{(\sigma_1-\sigma_3)/2}{(\sigma_1+\sigma_3)/2} = \frac{\sigma_1-\sigma_3}{\sigma_1+\sigma_3} = \frac{q}{p}$$

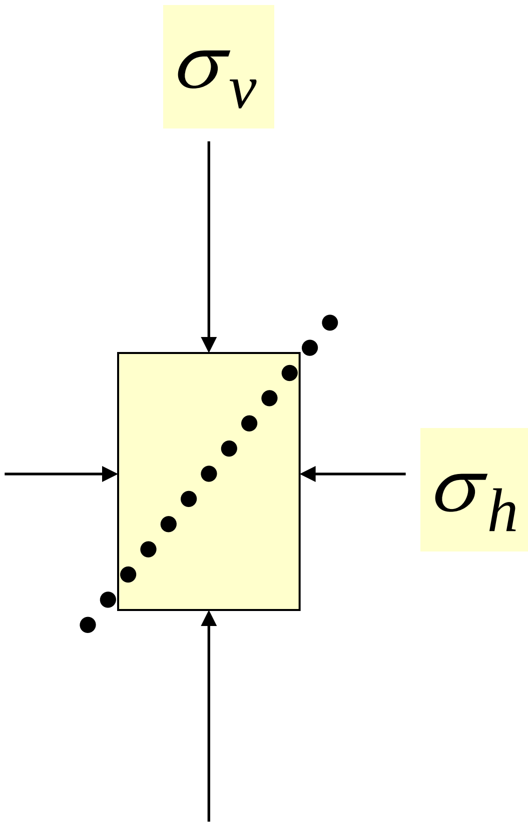
$$= \frac{\sigma_1/\sigma_3 - 1}{\sigma_1/\sigma_3 + 1} = \frac{1-\sigma_3/\sigma_1}{1+\sigma_3/\sigma_1}$$

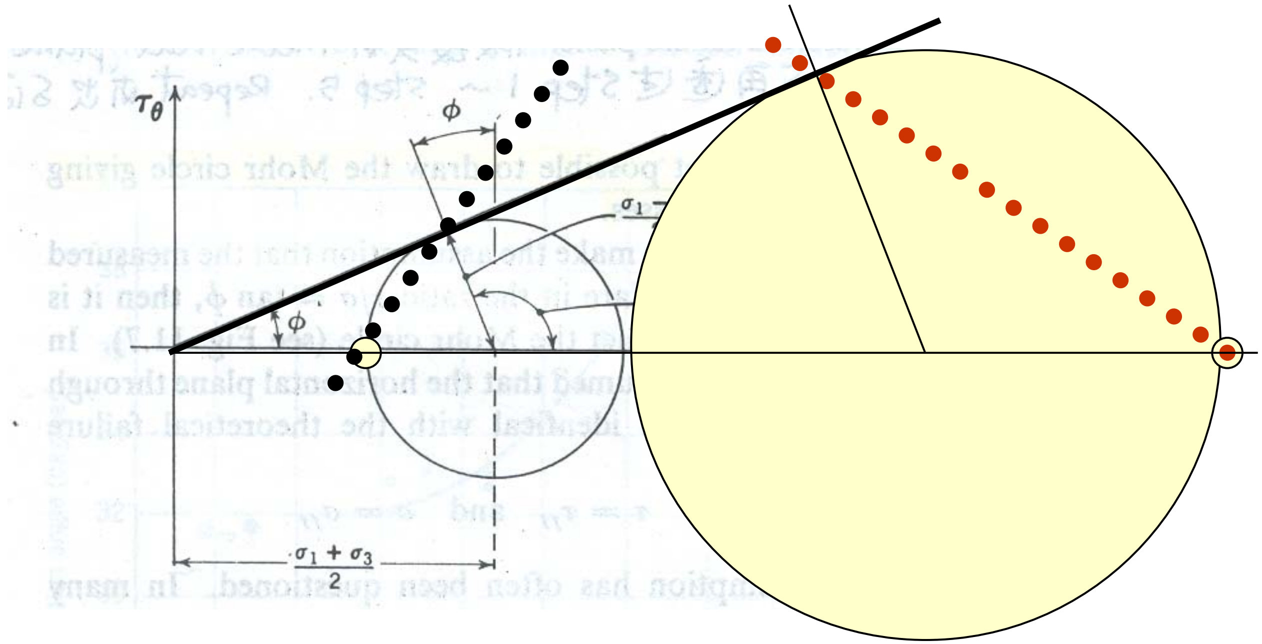
$$\frac{\sigma_1}{\sigma_3} = \frac{1+\sin\phi}{1-\sin\phi}$$

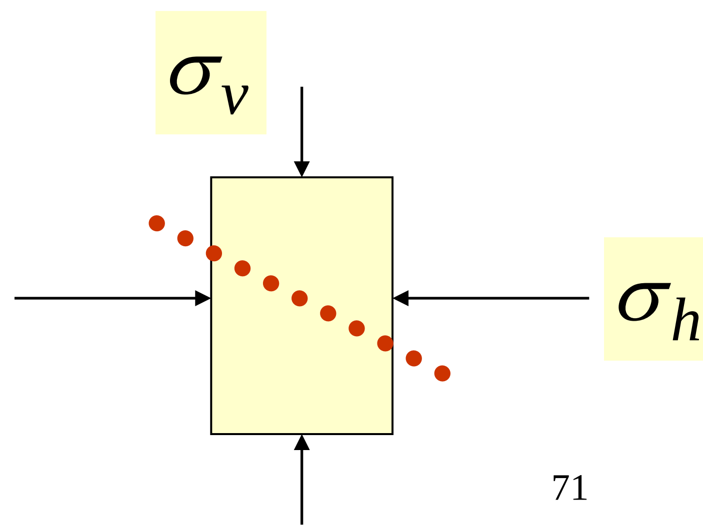
$$= \tan^2(45° + \phi/2) = \tan^2\theta_{cr}$$

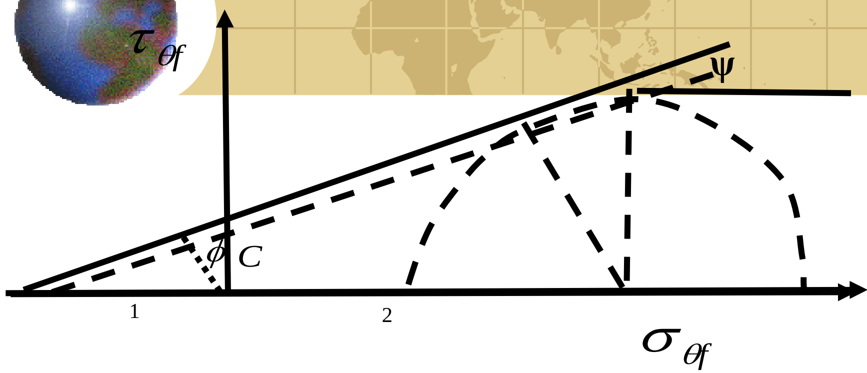
Note. For convenience, subscript f has been omitted from σ_{1f} and σ_{3f}.

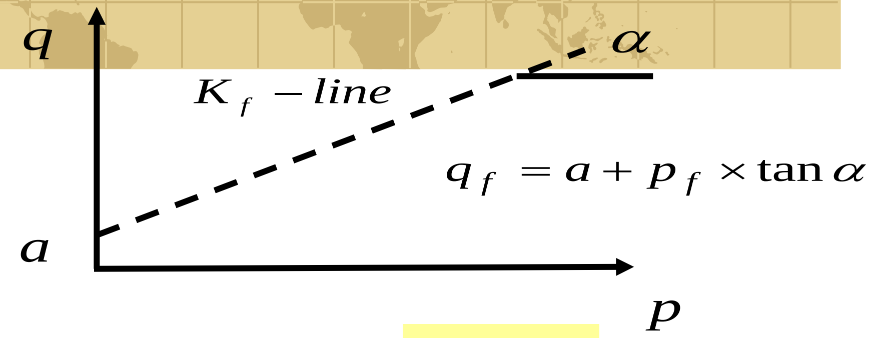
若假設破壞準則符合摩爾庫倫破壞準則(直線破壞包絡線)破壞面是哪一個?與水平面之夾角?











$\sin\phi = \tan\alpha$

$$c = \frac{a}{\cos\phi}$$

Prof: 1 $(1+2)\times\sin\phi = (1+2)\times\tan\alpha$

$$\therefore \sin\phi = \tan\alpha$$

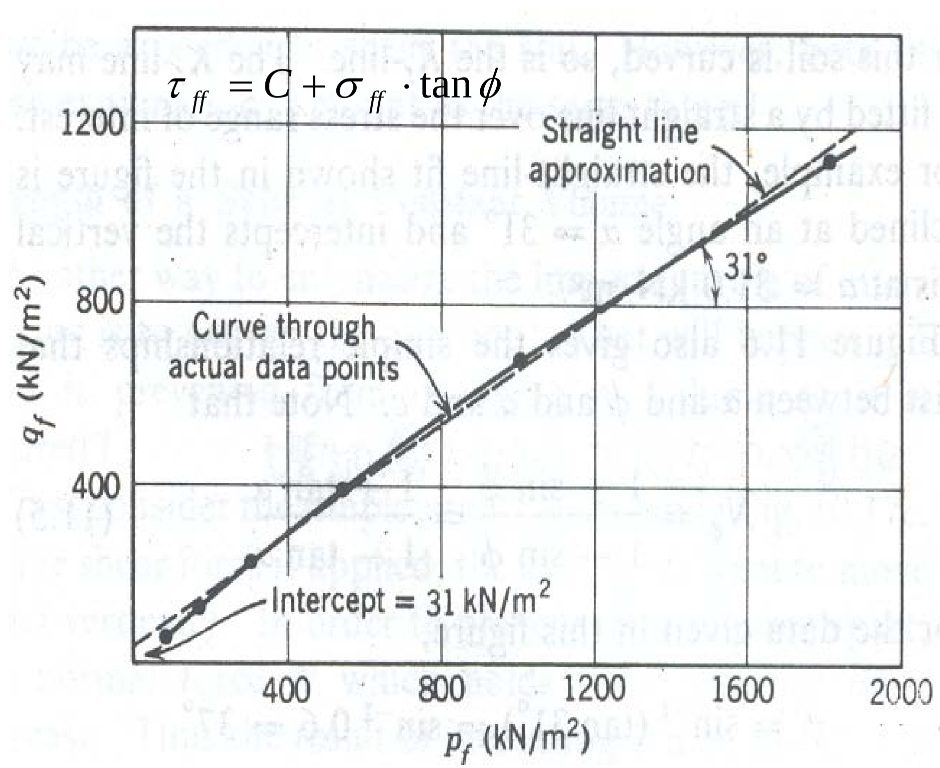
2 $$\frac{R}{a} = \frac{1+2}{1} = \frac{R}{c\cdot\cos\phi}$$

$$\therefore a = c\cdot\cos\phi \qquad c = \frac{a}{\cos\phi}$$

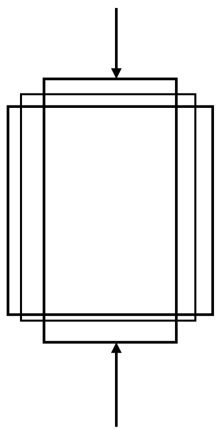
理論破壞面 $45^\circ + \dfrac{\phi}{2}$

觀察剪應變集中之面

兩面不相同，但一般相差不超過5°



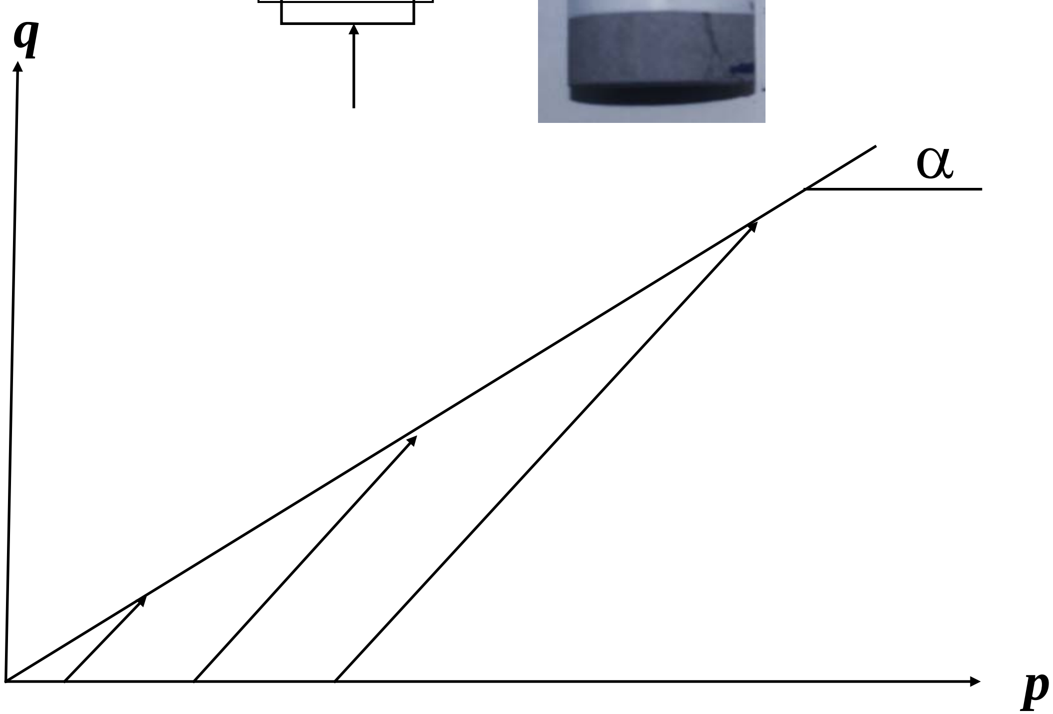
HW5



$\sigma_3 = 50kPa \quad \sigma_{1,f} = 150kPa$

$\sigma_3 = 150kPa \quad \sigma_{1,f} = 450kPa$

$\sigma_3 = 250kPa \quad \sigma_{1,f} = 750kPa$



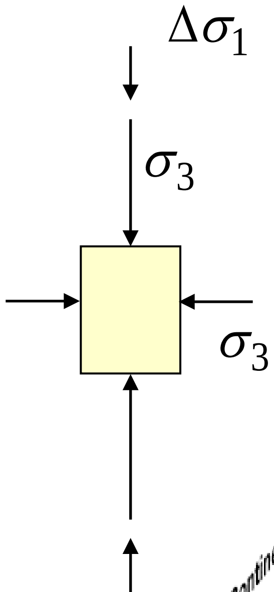
Find:
Friction angle and
orientation of failure surface

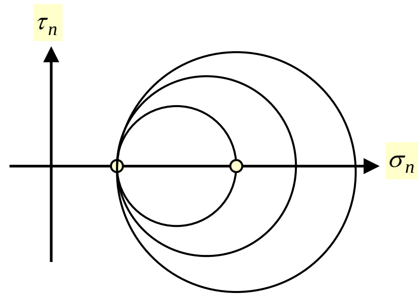
1 Stress and infinitesimal strain

Topic 1 Stress

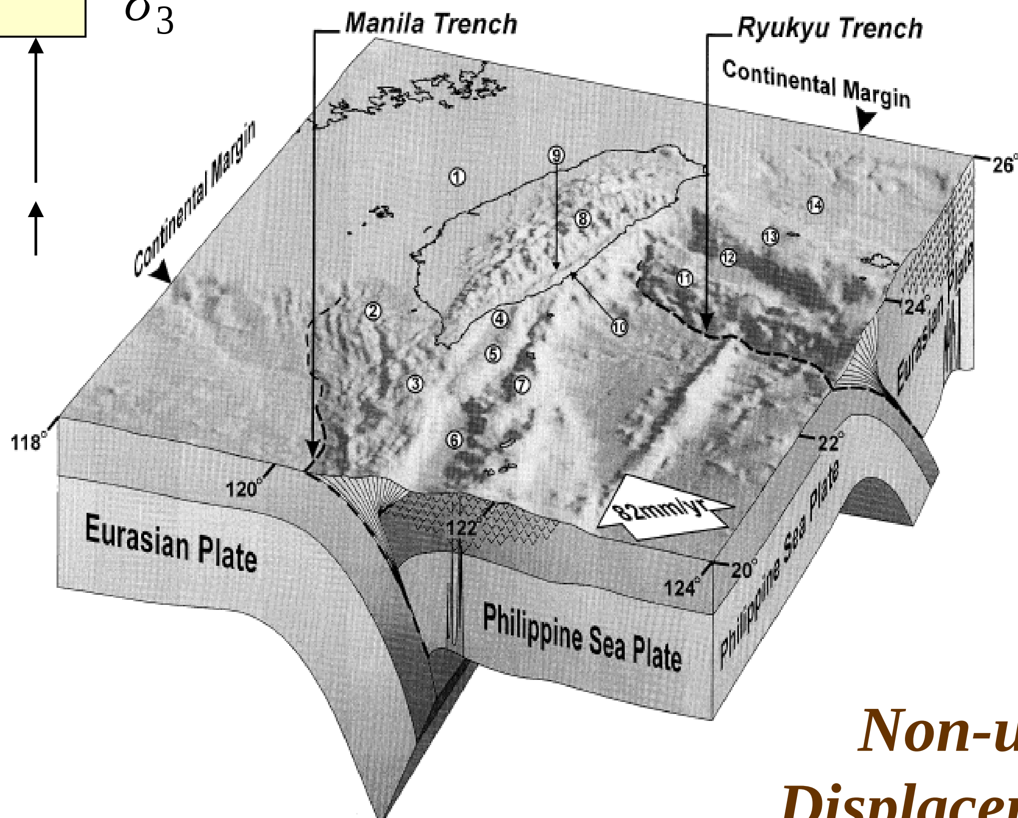
Topic 2 Strain

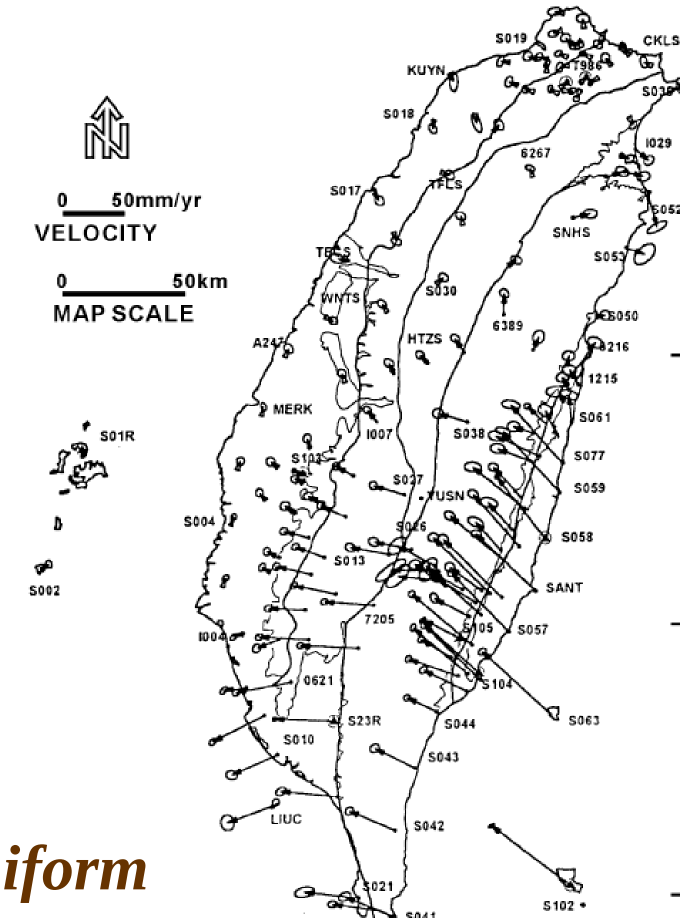
Topic 3 Elastic constants





- 菲律賓海板塊與歐亞大陸板塊聚合速率~8cm/year



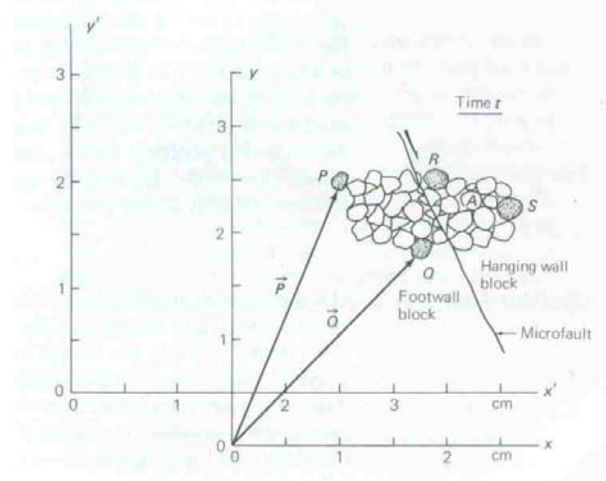


Non-uniform Displacement Field

Change in Mechanical State

- 一個狀態到另一個狀態之變化
  - Displacement (position)
  - Strains (configuration)

Displacement



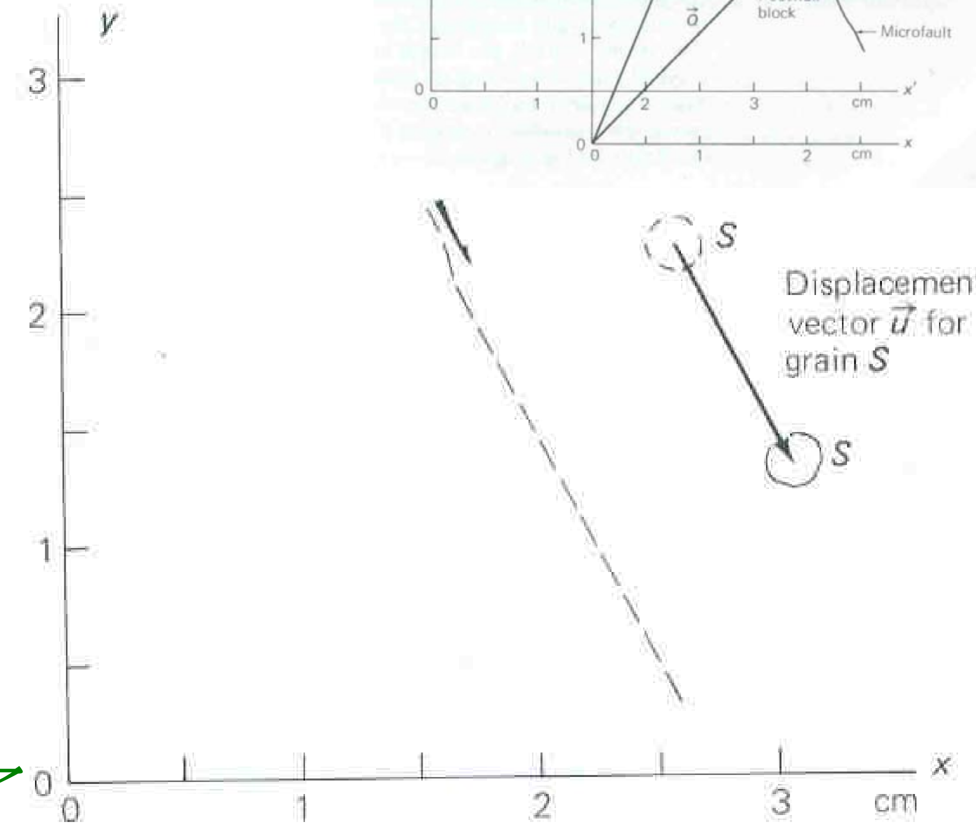


Figure 3.1 Displacement vector for grain *S* in Figure 2.1. Dashed outline is position of grain at time *t*; solid outline is position at time *t* + 1 yr.

t+1 years S的位置改變了

Displacement

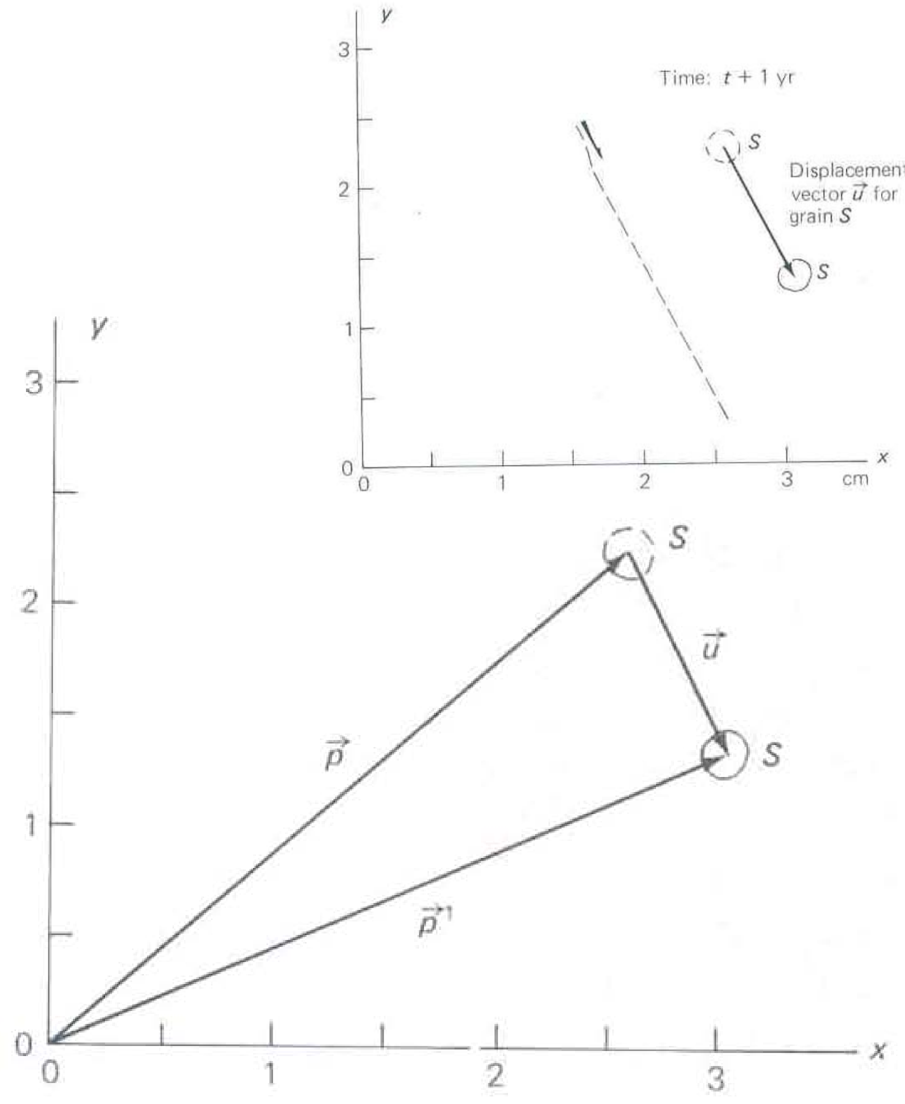
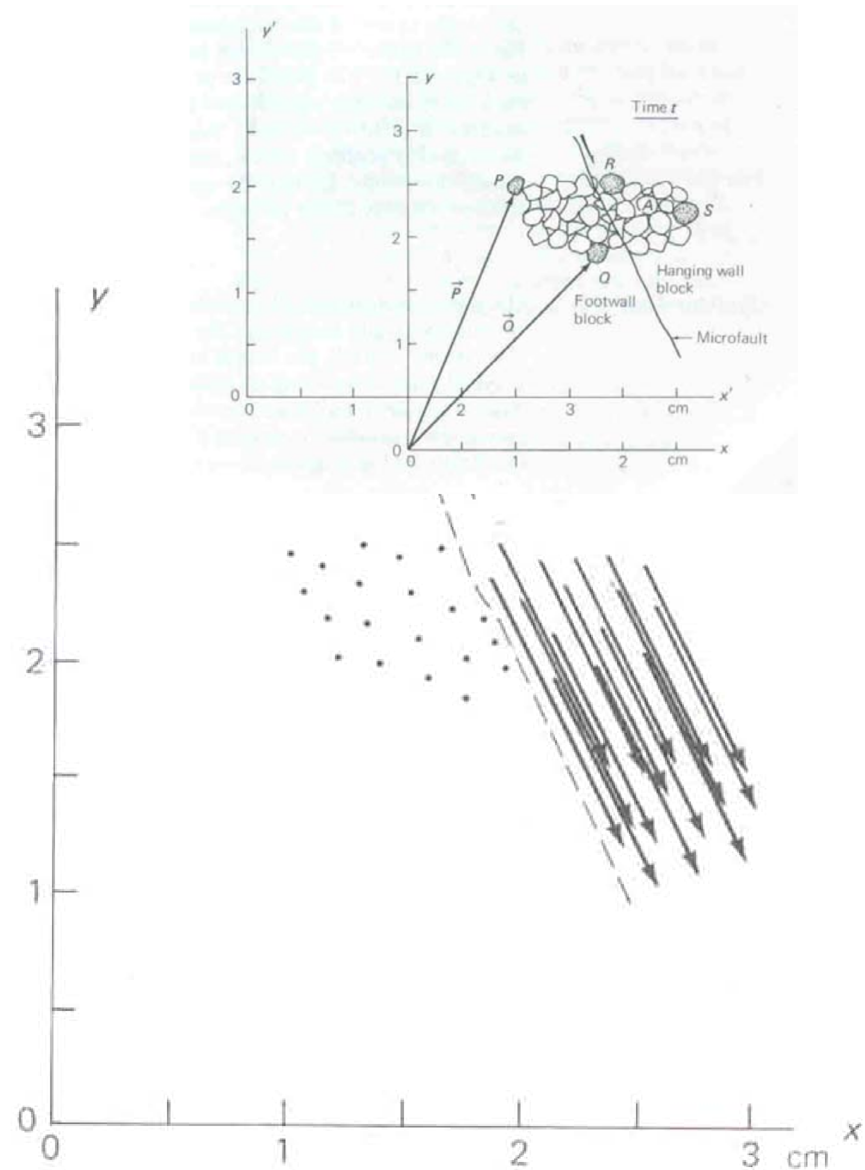


Figure 3.2 Position vectors of grain S in Figure 2.1 at time t ($\vec{p}$) and at time $t + 1$ yr ($\vec{p}^{\,1}$). The displacement vector ($\vec{u}$) connects the head of $\vec{p}$ (the old position of the grain) to the head of $\vec{p}^{\,1}$ (the new position of the grain).

Displacement

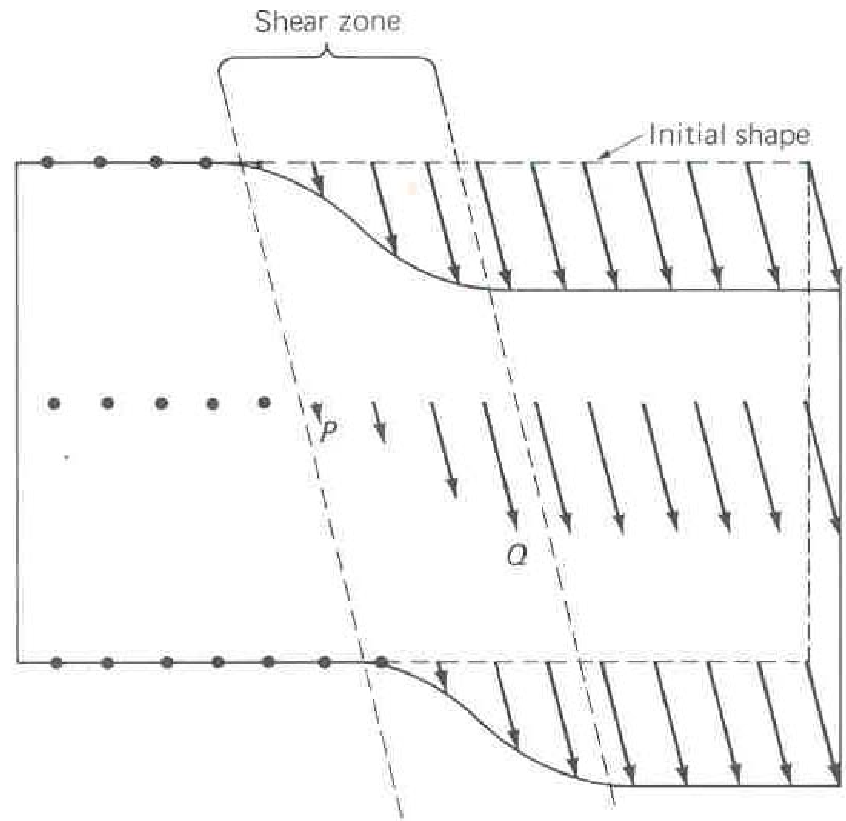
Figure 3.3 The displacement field for all the grains shown in Figure 2.1, indicating the change in position of each grain between time *t* and time *t* + 1 yr. Notice that the displacement vectors for grains in the footwall block plot as dots only. They are null vectors because the reference frame is arbitrarily fixed to the footwall block, and the grains have not moved in this reference frame.



思考：均勻位移場的意義為何？

Displacement

Figure 3.5 Displacement field associated with a shear zone in a block of rock. Initial shape of block is indicated.



剪力帶
非均勻位移場

→ **Strain**

Displacement

非均勻位移場

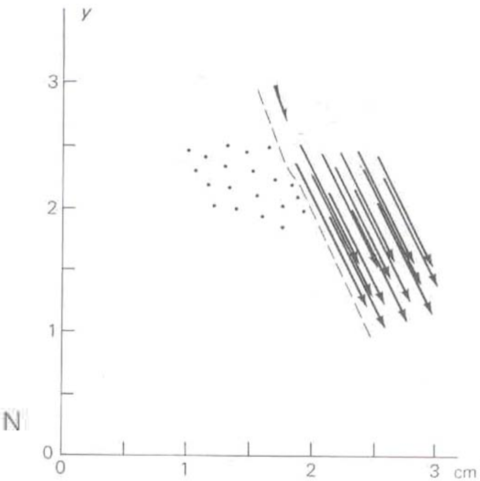
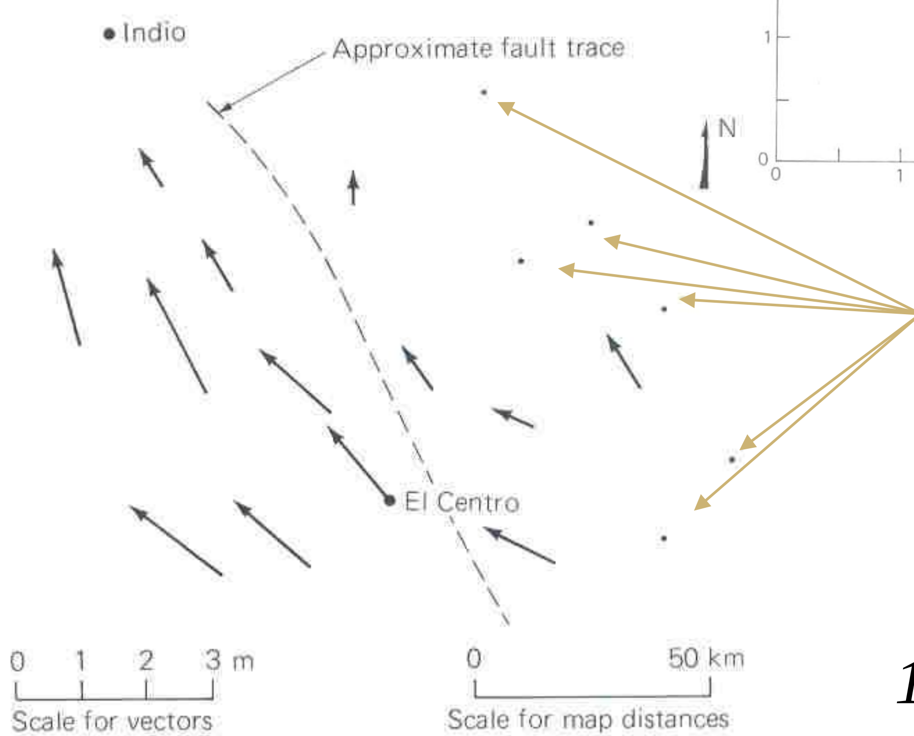


Figure 3.4 Displacement field for survey markers either side of a fault in the Imperial Valley, California. Each displacement vector represents the change in position of a station between triangulation surveys of 1941 and 1954. The reference frame is fixed to six markers in the eastern part of the map that show no displacement. (After Whitten, 1956, Figure 3).



Fix point

1941-1954

Imperial Valley, California

Strains

1. 兩點間距離是不是改變了？
2. Configuration是不是改變了？

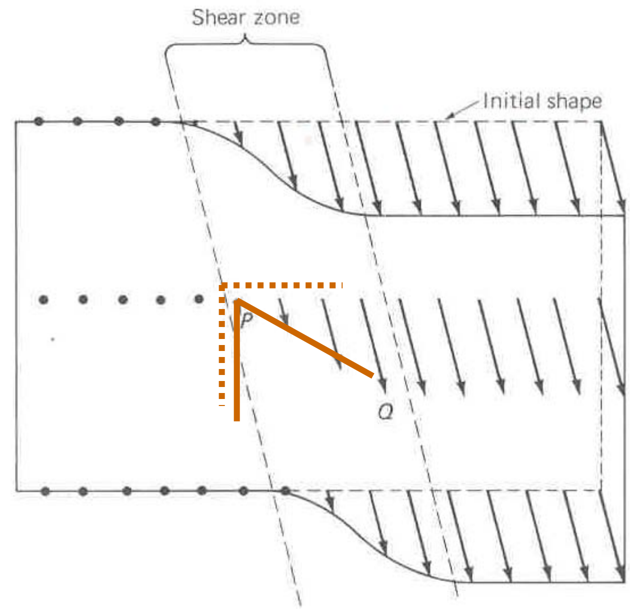
PQ長度變長了(正向應變)

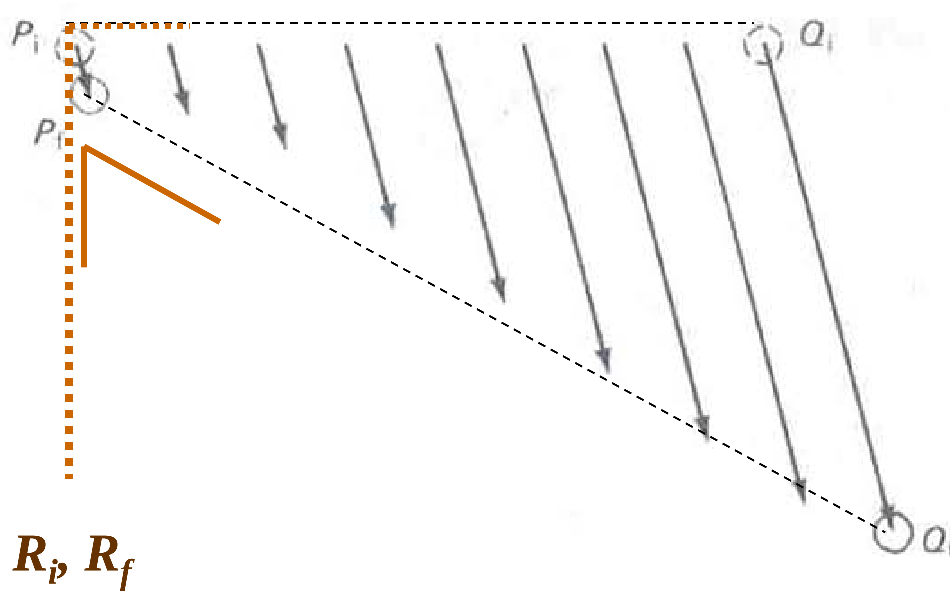
PQ與PR 間夾角變小了(剪應變)

$\varepsilon = (P_f Q_f - P_i Q_i) / (P_i Q_i)$

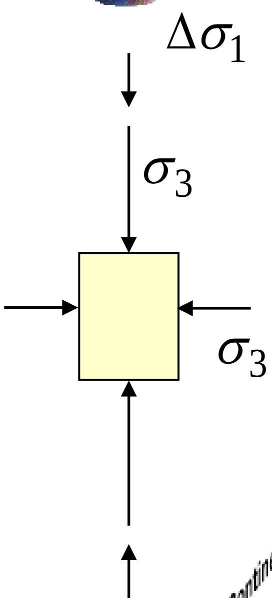
$\gamma = \angle Q_i P_i R_i - \angle Q_f P_f R_f$

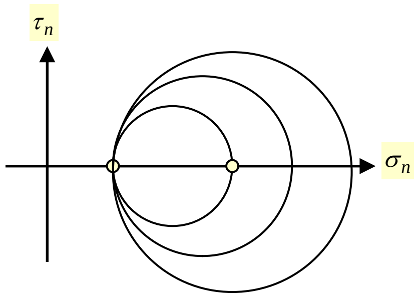
Figure 3.6 Enlarged central part of displacement field of Figure 3.5, showing initial (P_i, Q_i) and final (P_f, Q_f) positions of grains P and Q.

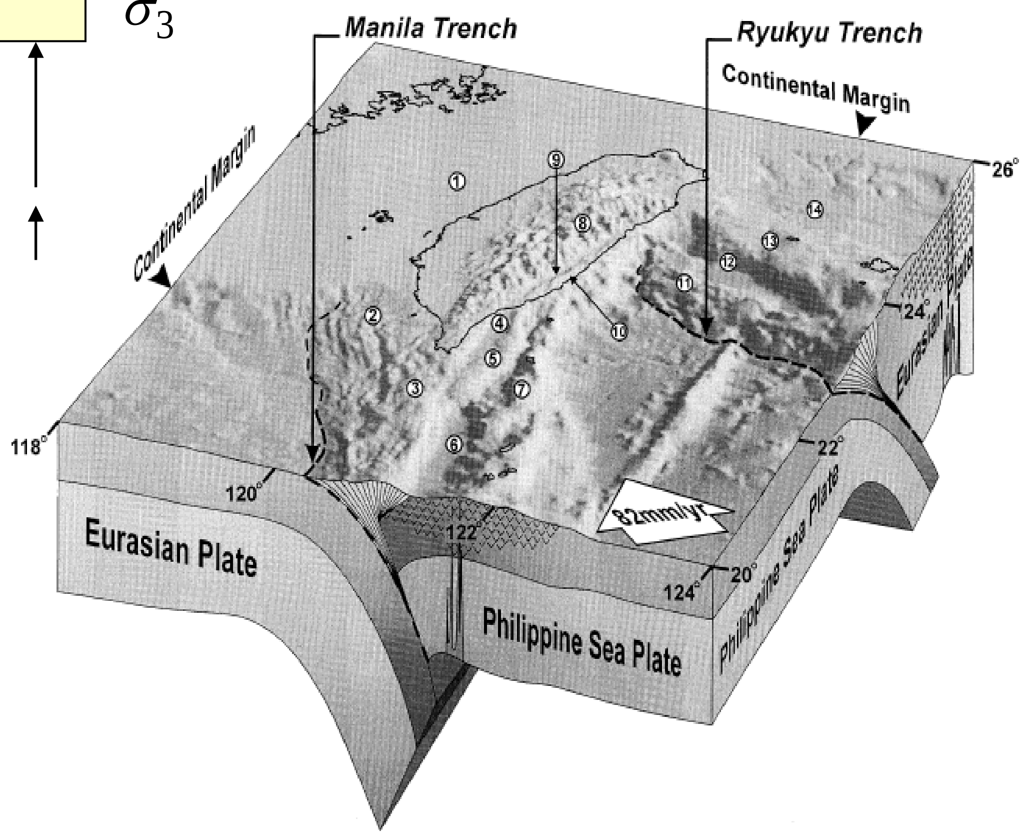


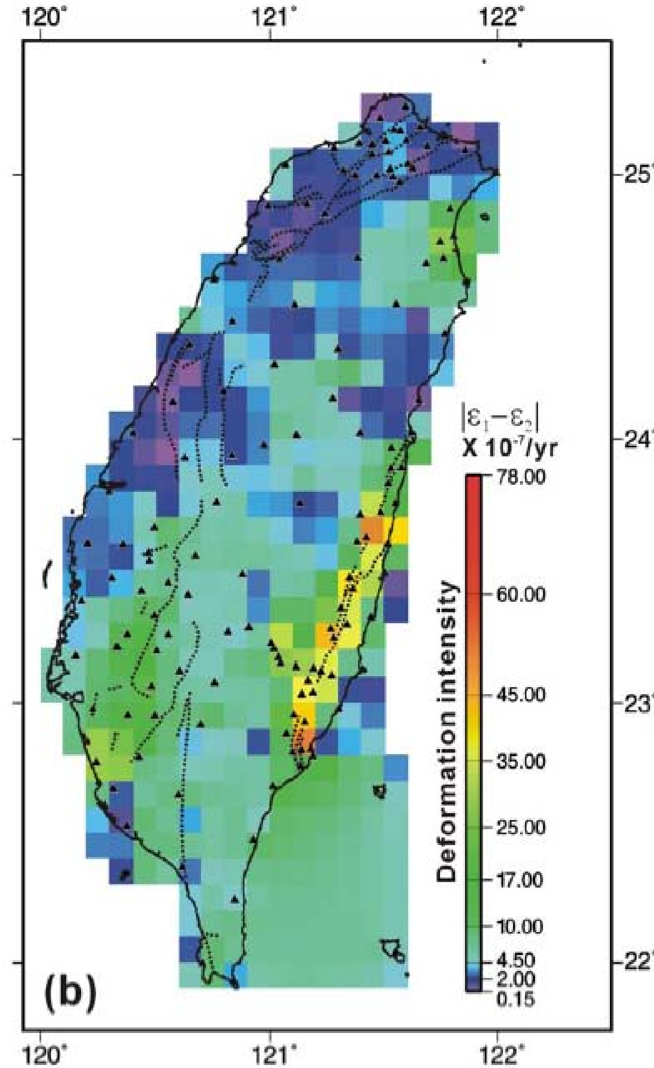


菲律賓海板塊與歐亞大陸板塊聚合速率~8cm/year





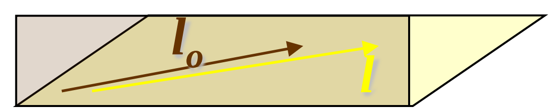




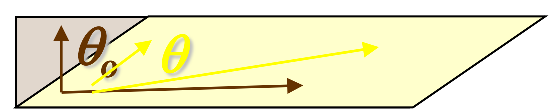
What is strain

- **Totality of all changes in length of fibers of the material which pass through the point & changes in the angle between any pair of lines radiating from this point.**

Linear strain $$\varepsilon_n = \frac{l - l_o}{l_o}$$

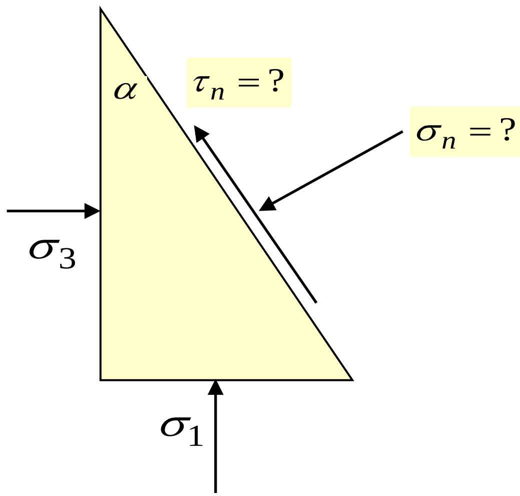


Shear strain $$\gamma_n = \theta - \theta_0$$



Two dimensional strain analysis

Stress analysis



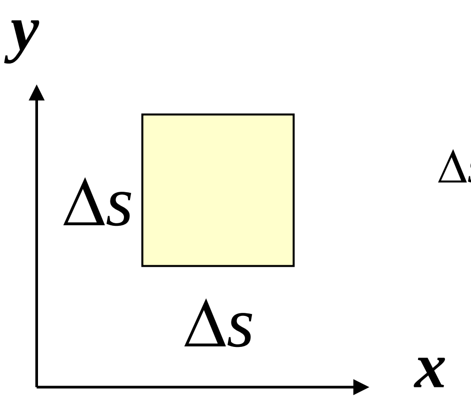
$$\begin{cases} \sigma_n = \frac{1}{2}(\sigma_1+\sigma_3) - \frac{1}{2}(\sigma_1-\sigma_3)\times\cos 2\alpha \\ \tau_n = \qquad\qquad\qquad -\frac{1}{2}(\sigma_1-\sigma_3)\times\sin 2\alpha \end{cases}$$

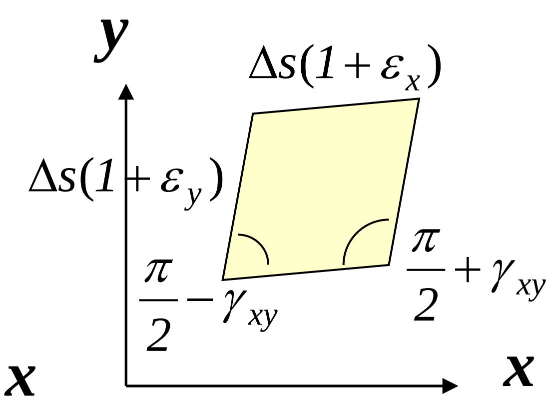
$$(x-x_o)^2 + y^2 = R^2$$

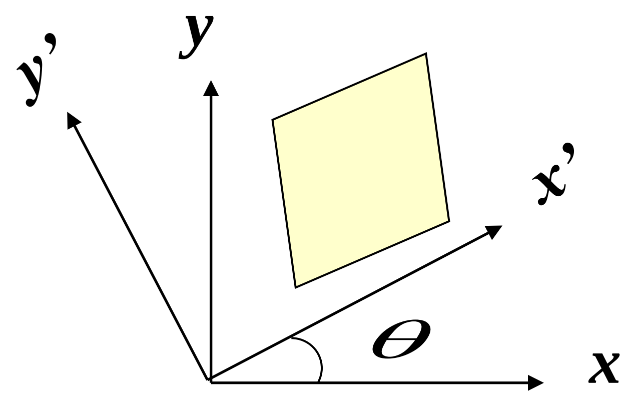
Initial state

Final state

Linear strain $\varepsilon_{x'}, \varepsilon_{y'}$ ***and Shear strain*** $\gamma_{x'y'}$ ***in x'-y' coordinate ?***

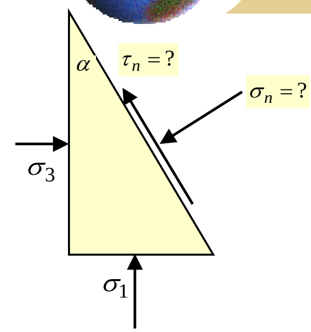




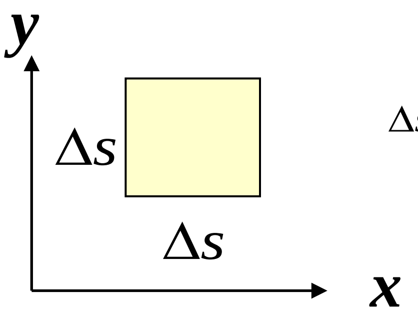


Strain analysis

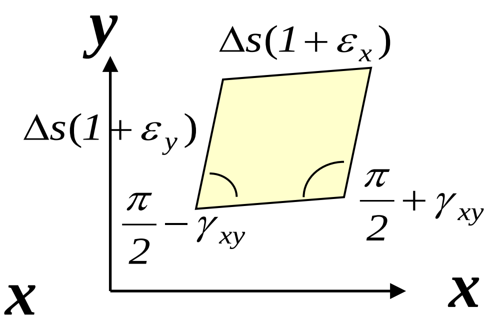
Two dimensional strain analysis



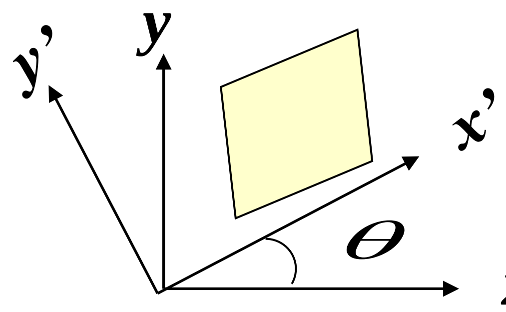
Initial state

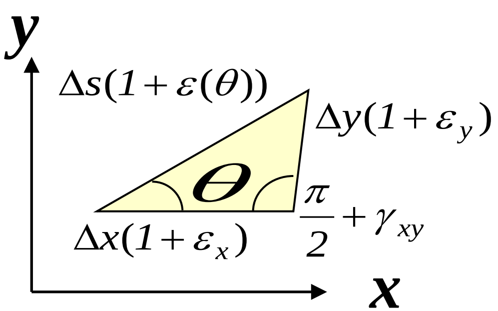


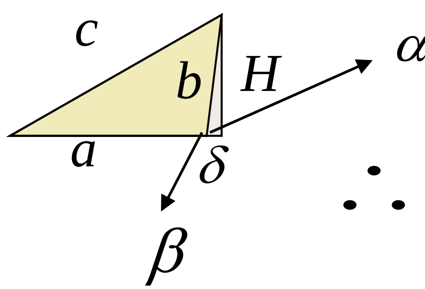
Final state



Linear strain and Shear strain in x'-y' coordinate ?



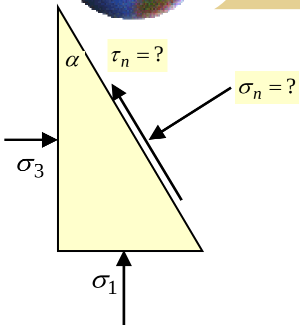




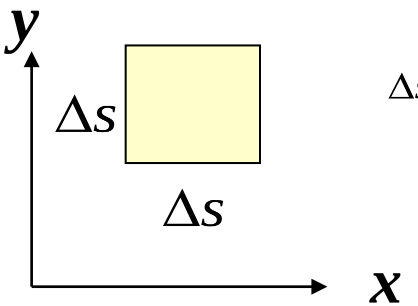
$$
\begin{aligned}
a^2+b^2 &= a^2+H^2+\delta^2 \\
&= (a+\delta)^2+H^2-2\cdot a\cdot\delta \\
&= c^2-2\cdot a\cdot b\cdot\cos\alpha
\end{aligned}
$$

$$
\begin{aligned}
\therefore c^2 &= a^2+b^2+2\cdot a\cdot b\cdot\cos\alpha \\
&= a^2+b^2-2\cdot a\cdot b\cdot\cos\beta \\
&= a^2+b^2-2\cdot a\cdot b\cdot\cos\left(\frac{\pi}{2}+\gamma_{xy}\right)
\end{aligned}
$$

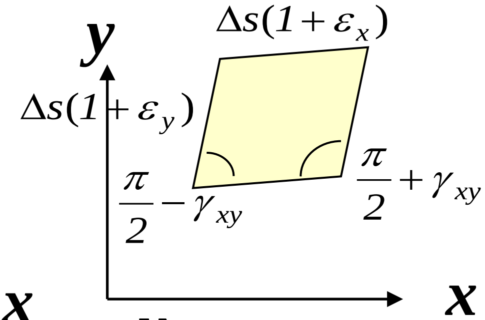
Two dimensional strain analysis



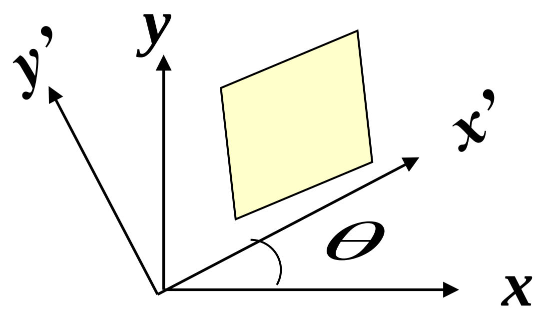
Initial state

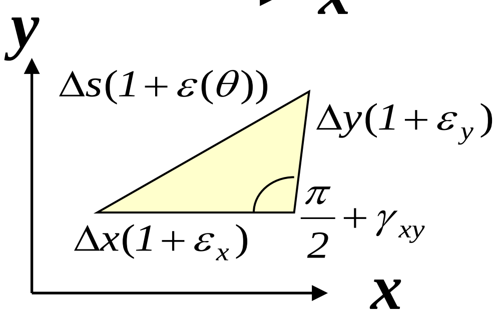


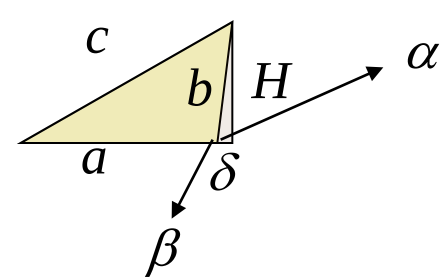
Final state



Linear strain and Shear strain in x'-y' coordinate ?







$$c^2 = a^2 + b^2 - 2\cdot a\cdot b\cdot\cos\left(\frac{\pi}{2}+\gamma_{xy}\right)$$

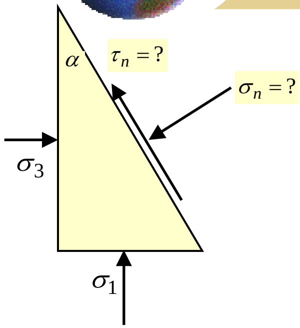
$$(\Delta s)^2\cdot(1+\varepsilon(\theta))^2 = (\Delta x)^2\cdot((1+\varepsilon_x))^2 + (\Delta y)^2\cdot((1+\varepsilon_y))^2 - 2\cdot(\Delta x)\cdot(1+\varepsilon_x)\cdot(\Delta y)\cdot(1+\varepsilon_x)\cdot\cos\left(\frac{\pi}{2}+\gamma_{xy}\right)$$

$\because$

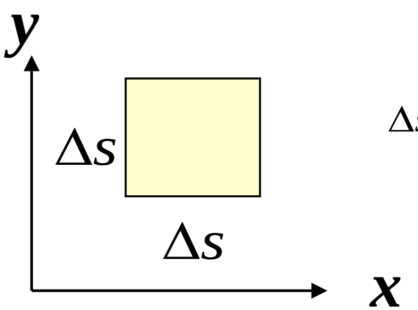
1. $\Delta x = \Delta s\cdot\cos\theta, \Delta y = \Delta s\cdot\cos\theta$
2. $\cos\left(\frac{\pi}{2}+\gamma_{xy}\right) = -\sin\gamma_{xy} \cong -\gamma_{xy}$
3. 消去高次項

$$\varepsilon(\theta) = \varepsilon_x\cdot\cos^2\theta + \varepsilon_y\cdot\sin^2\theta + \gamma_{xy}\cdot\sin\theta\cdot\cos\theta$$
$$= \varepsilon_x\cdot\left(\frac{1+\cos 2\theta}{2}\right) + \varepsilon_y\cdot\left(\frac{1-\cos 2\theta}{2}\right) + \frac{1}{2}\cdot\gamma_{xy}\cdot\sin 2\theta$$

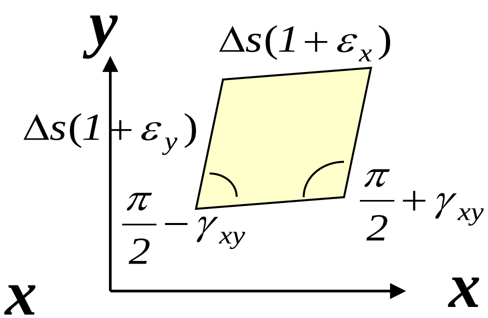
Two dimensional strain analysis



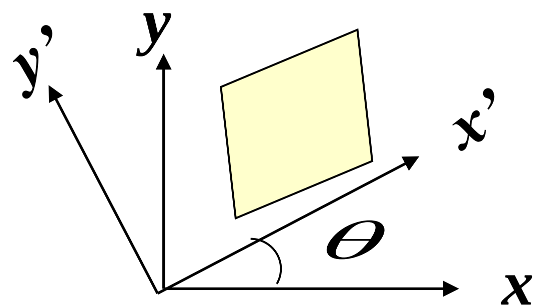
Initial state

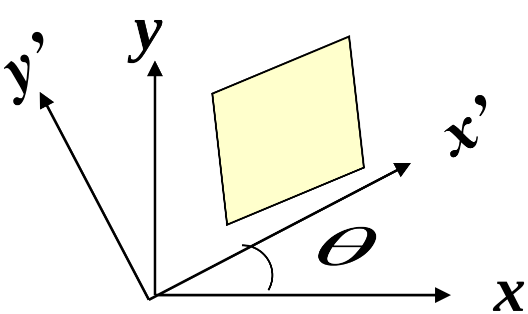


Final state



Linear strain and Shear strain in x'-y' coordinate ?





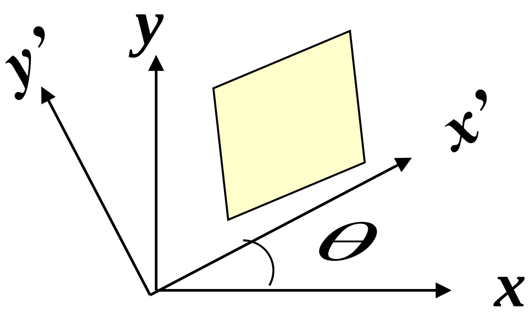
$$\varepsilon(\theta) = \varepsilon_x \cdot \cos^2\theta + \varepsilon_y \cdot \sin^2\theta + \gamma_{xy} \cdot \sin\theta \cdot \cos\theta$$

$$= \varepsilon_x \cdot \left(\frac{1+\cos 2\theta}{2}\right) + \varepsilon_y \cdot \left(\frac{1-\cos 2\theta}{2}\right) + \frac{1}{2} \cdot \gamma_{xy} \cdot \sin 2\theta$$

$$\varepsilon_{x'} = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) + \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right) \cdot \cos 2\theta + \frac{\gamma_{xy}}{2} \cdot \sin 2\theta$$

$$\varepsilon_{y'} = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) - \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right) \cdot \cos 2\theta - \frac{\gamma_{xy}}{2} \cdot \sin 2\theta$$

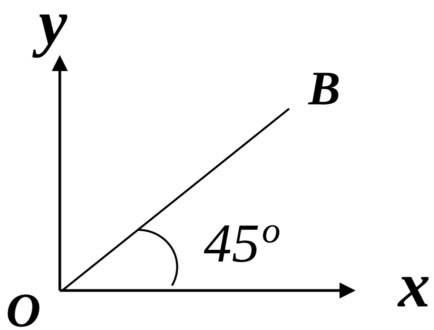
Two dimensional strain analysis



$$\varepsilon(\theta) = \varepsilon_x \cdot \cos^2\theta + \varepsilon_y \cdot \sin^2\theta + \gamma_{xy} \cdot \sin\theta \cdot \cos\theta$$

$$= \varepsilon_x \cdot (\frac{1+\cos 2\theta}{2}) + \varepsilon_y \cdot (\frac{1-\cos 2\theta}{2}) + \frac{1}{2} \cdot \gamma_{xy} \cdot \sin 2\theta$$

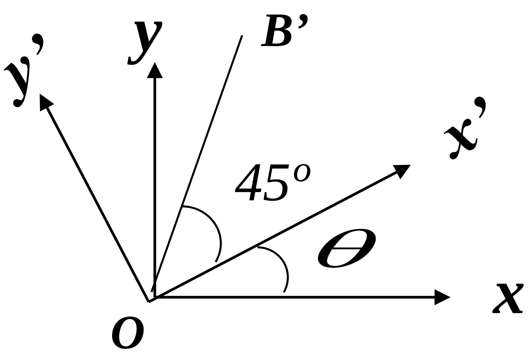
$$\varepsilon_{x'} = (\frac{\varepsilon_x + \varepsilon_y}{2}) + (\frac{\varepsilon_x - \varepsilon_y}{2}) \cdot \cos 2\theta + \frac{\gamma_{xy}}{2} \cdot \sin 2\theta \qquad \textbf{Eq.(1)}$$



$$\varepsilon_{y'} = (\frac{\varepsilon_x + \varepsilon_y}{2}) - (\frac{\varepsilon_x - \varepsilon_y}{2}) \cdot \cos 2\theta - \frac{\gamma_{xy}}{2} \cdot \sin 2\theta \qquad \textbf{Eq.(2)}$$

$$\varepsilon_{OB} = \varepsilon(45^o) = \frac{1}{2} \cdot (\varepsilon_x + \varepsilon_y + \gamma_{xy})$$

$$\gamma_{xy} = 2 \cdot \varepsilon_{OB} - (\varepsilon_x + \varepsilon_y)$$



$$\gamma_{x'y'} = 2 \cdot \varepsilon_{OB'} - (\varepsilon_{x'} + \varepsilon_{y'}) \qquad \textbf{Eq.(3)}$$

$$\varepsilon_{OB'} = (\frac{\varepsilon_x + \varepsilon_y}{2}) + (\frac{\varepsilon_x - \varepsilon_y}{2}) \cdot \cos 2 \cdot (\theta + 45^o) + \frac{\gamma_{xy}}{2} \cdot \sin 2 \cdot (\theta + 45^o)$$

$$= (\frac{\varepsilon_x + \varepsilon_y}{2}) - (\frac{\varepsilon_x - \varepsilon_y}{2}) \cdot \sin 2\theta + \frac{\gamma_{xy}}{2} \cdot \cos 2\theta \qquad \textbf{Eq.(4)}$$

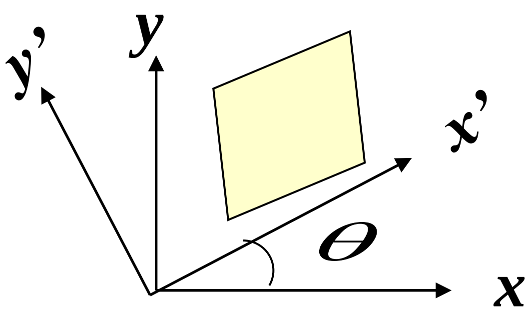
Inser Eq.(1),(2),(4) into Eq.(3)

$$\frac{\gamma_{x'y'}}{2} = -(\frac{\varepsilon_x - \varepsilon_y}{2}) \cdot \sin 2\theta + \frac{\gamma_{xy}}{2} \cdot \cos 2\theta$$

Two dimensional strain analysis

$$\varepsilon(\theta) = \varepsilon_x \cdot \cos^2\theta + \varepsilon_y \cdot \sin^2\theta + \gamma_{xy} \cdot \sin\theta \cdot \cos\theta$$

$$= \varepsilon_x \cdot \left(\frac{1+\cos 2\theta}{2}\right) + \varepsilon_y \cdot \left(\frac{1-\cos 2\theta}{2}\right) + \frac{1}{2} \cdot \gamma_{xy} \cdot \sin 2\theta$$



$$\varepsilon_{x'} = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) + \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right) \cdot \cos 2\theta + \frac{\gamma_{xy}}{2} \cdot \sin 2\theta$$

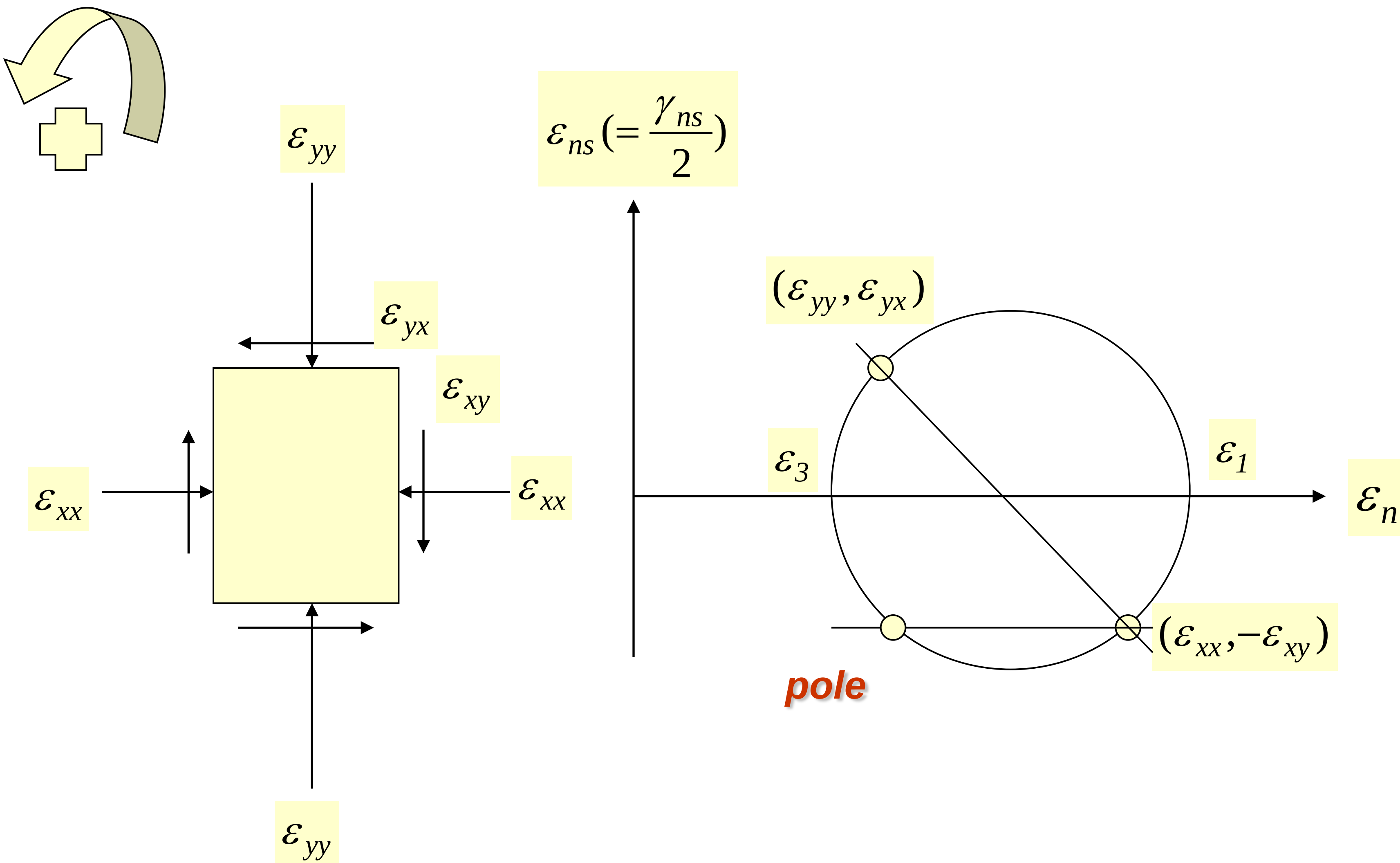
$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right) \cdot \sin 2\theta + \frac{\gamma_{xy}}{2} \cdot \cos 2\theta$$

$$x = x_o + A \cdot \cos 2\theta + B \cdot \sin 2\theta$$

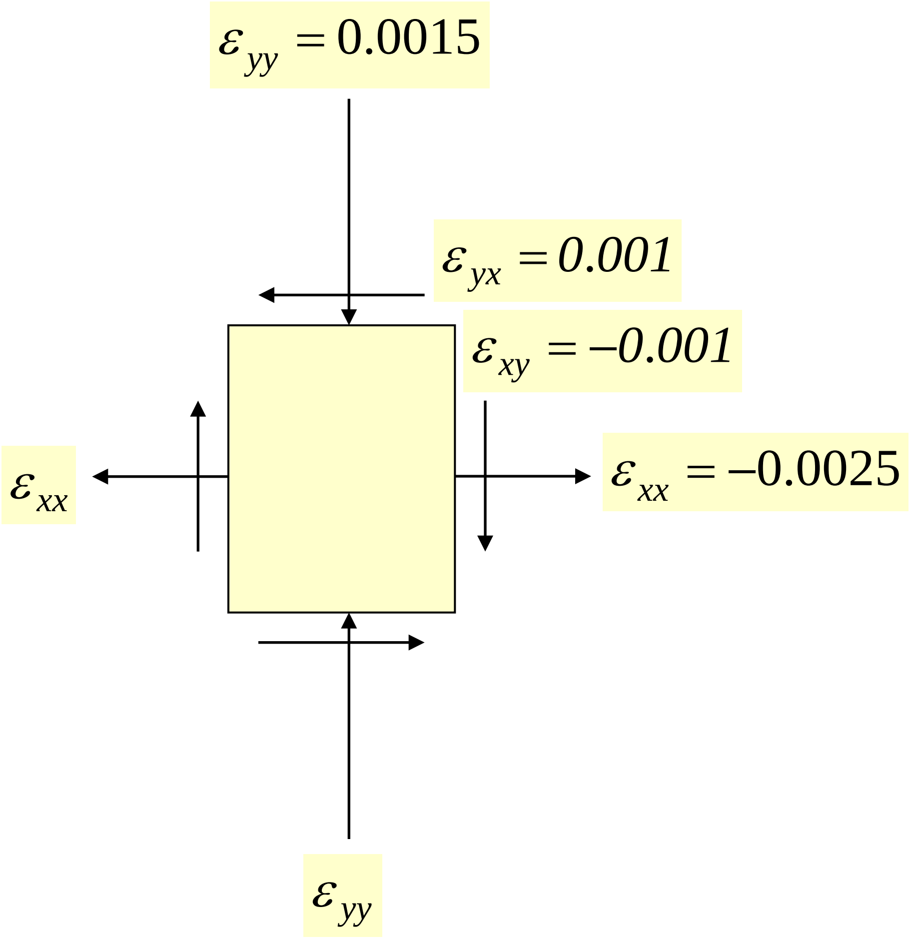
$$y = -A \cdot \sin 2\theta + B \cdot \cos 2\theta$$

$$(x - x_o)^2 + y^2 = R^2$$

Construct a Mohr circle for strain

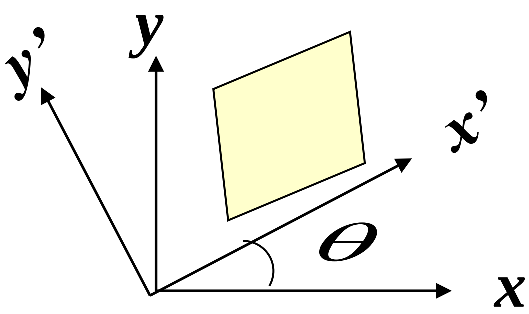


HW6



1. Plot Mohr's circle for strains for this state.
2. Find the magnitudes and directions of the principal strains from the circle.
3. Find the maximum shear strain from the circle.

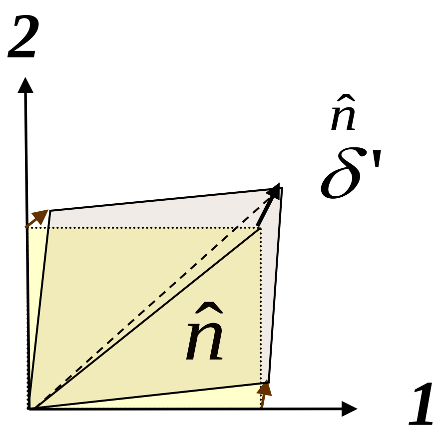
Two dimensional strain analysis

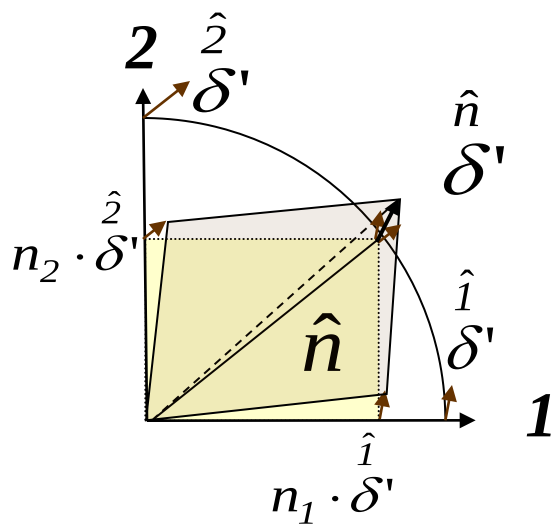


$$\varepsilon_{x'} = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) + \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\cdot \cos 2\theta + \frac{\gamma_{xy}}{2}\cdot \sin 2\theta$$

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\cdot \sin 2\theta + \frac{\gamma_{xy}}{2}\cdot \cos 2\theta$$

Look in different way!

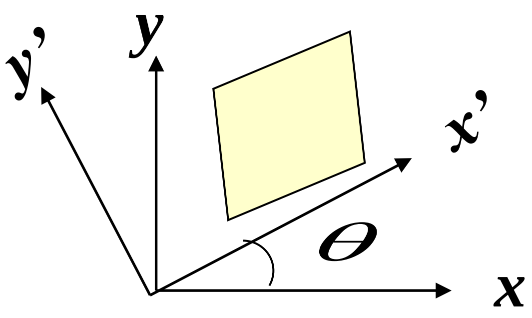




Relative displacement of unit length

$$\overset{\hat{n}}{\delta'} = n_1 \cdot \overset{\hat{1}}{\delta'} + n_2 \cdot \overset{\hat{2}}{\delta'}$$

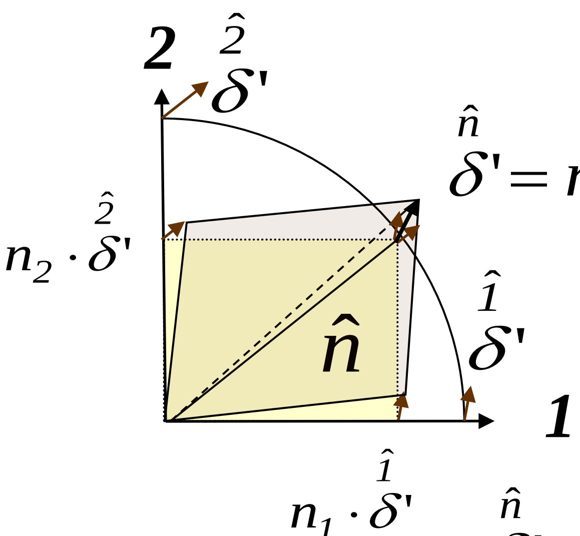
Two dimensional strain analysis



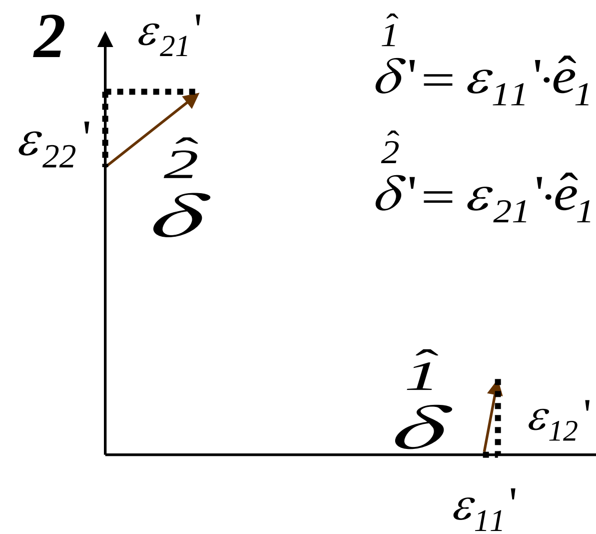
$$\varepsilon_{x'} = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) + \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\cdot\cos 2\theta + \frac{\gamma_{xy}}{2}\cdot\sin 2\theta$$

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\cdot\sin 2\theta + \frac{\gamma_{xy}}{2}\cdot\cos 2\theta$$

Look in different way!



$$\overset{\hat{n}}{\delta}' = n_1 \cdot \overset{\hat{1}}{\delta}' + n_2 \cdot \overset{\hat{2}}{\delta}'$$

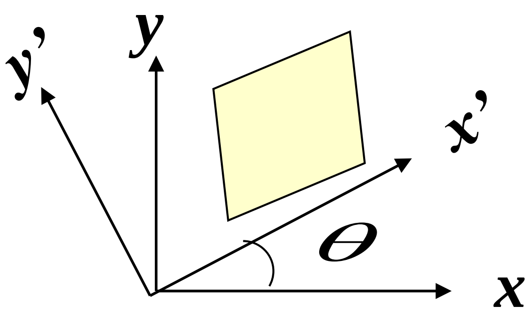


$$\overset{\hat{1}}{\delta}' = \varepsilon_{11}'\cdot\hat{e}_1 + \varepsilon_{12}'\cdot\hat{e}_2$$

$$\overset{\hat{2}}{\delta}' = \varepsilon_{21}'\cdot\hat{e}_1 + \varepsilon_{22}'\cdot\hat{e}_2$$

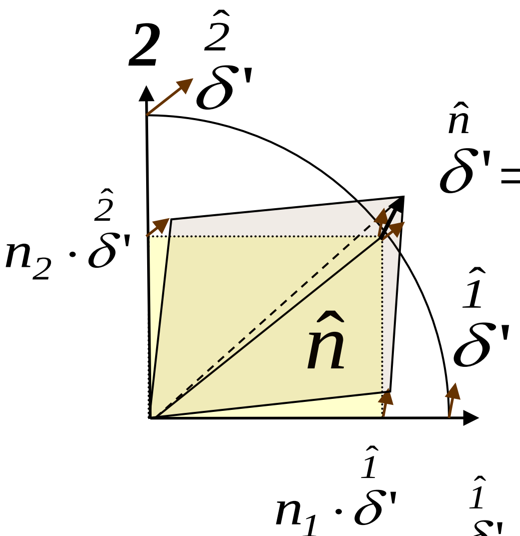
$$\overset{\hat{n}}{\delta}' = (n_1\cdot\varepsilon_{11}' + n_2\cdot\varepsilon_{21}')\cdot\hat{e}_1 + (n_1\cdot\varepsilon_{12}' + n_2\cdot\varepsilon_{22}')\cdot\hat{e}_2$$

Three dimensional strain analysis



$$\varepsilon_{x'} = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) + \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\cdot\cos 2\theta + \frac{\gamma_{xy}}{2}\cdot\sin 2\theta$$

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\cdot\sin 2\theta + \frac{\gamma_{xy}}{2}\cdot\cos 2\theta$$



$$\overset{\hat{n}}{\delta}' = n_1 \cdot \overset{\hat{1}}{\delta}' + n_2 \cdot \overset{\hat{2}}{\delta}'$$

$$\overset{\hat{1}}{\delta}' = \varepsilon_{11}'\cdot\hat{e}_1 + \varepsilon_{12}'\cdot\hat{e}_2$$

$$\overset{\hat{2}}{\delta}' = \varepsilon_{21}'\cdot\hat{e}_1 + \varepsilon_{22}'\cdot\hat{e}_2$$

$$\overset{\hat{1}}{\delta}' = \varepsilon_{11}'\cdot\hat{e}_1 + \varepsilon_{12}'\cdot\hat{e}_2 + \varepsilon_{13}'\cdot\hat{e}_3$$

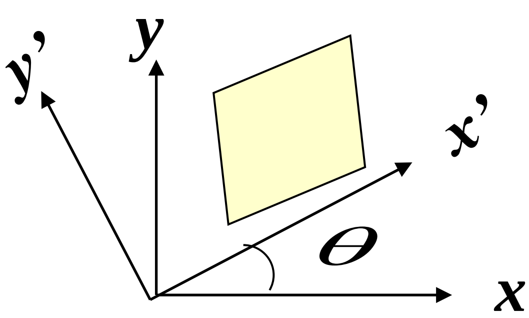
$$\overset{\hat{2}}{\delta}' = \varepsilon_{21}'\cdot\hat{e}_1 + \varepsilon_{22}'\cdot\hat{e}_2 + \varepsilon_{23}'\cdot\hat{e}_3$$

$$\overset{\hat{3}}{\delta}' = \varepsilon_{31}'\cdot\hat{e}_1 + \varepsilon_{32}'\cdot\hat{e}_2 + \varepsilon_{33}'\cdot\hat{e}_3$$

$$\begin{aligned}\overset{\hat{n}}{\delta}' = &(n_1\cdot\varepsilon_{11}' + n_2\cdot\varepsilon_{21}' + n_3\cdot\varepsilon_{31}')\cdot\hat{e}_1 \\ &+ (n_1\cdot\varepsilon_{12}' + n_2\cdot\varepsilon_{22}' + n_3\cdot\varepsilon_{32}')\cdot\hat{e}_2 \\ &+ (n_1\cdot\varepsilon_{13}' + n_2\cdot\varepsilon_{23}' + n_3\cdot\varepsilon_{33}')\cdot\hat{e}_3\end{aligned}$$

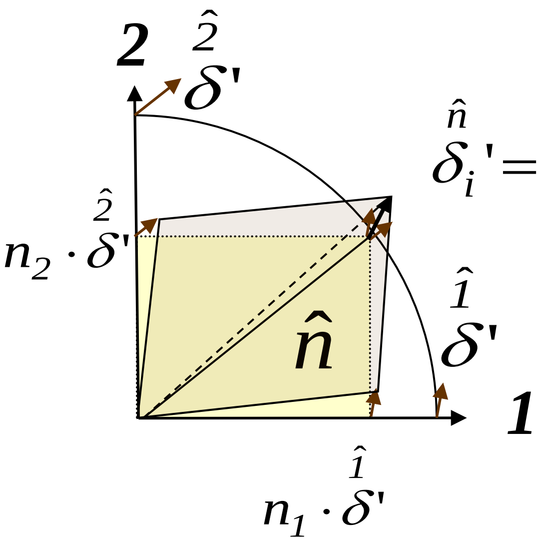
$$\overset{\hat{n}}{\delta}_i' = n_j \cdot \varepsilon_{ji}'$$

Three dimensional strain analysis



$$\varepsilon_{x'} = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) + \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\cdot \cos 2\theta + \frac{\gamma_{xy}}{2}\cdot \sin 2\theta$$

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\cdot \sin 2\theta + \frac{\gamma_{xy}}{2}\cdot \cos 2\theta$$



$$\overset{\hat{n}}{\delta}_i' = n_j \cdot \varepsilon_{ji}'$$

$$\varepsilon_{ij}' = \frac{1}{2}(\varepsilon_{ij}' + \varepsilon_{ji}') + \frac{1}{2}(\varepsilon_{ij}' - \varepsilon_{ji}')$$
$$= \varepsilon_{ij} + w_{ij}$$

$$\overset{\hat{n}}{\delta}_i' = n_j \cdot \varepsilon_{ji} + n_j \cdot w_{ji}$$
$$= \varepsilon_{ij} \cdot n_j - w_{ij} \cdot n_j$$
$$= \overset{\hat{n}}{\delta}_i + \overset{\hat{n}}{\Omega}_i$$

$$\overset{\hat{n}}{\delta}_i = \varepsilon_{ij} \cdot n_j$$

Pure deformation ***Rigid body rotation***