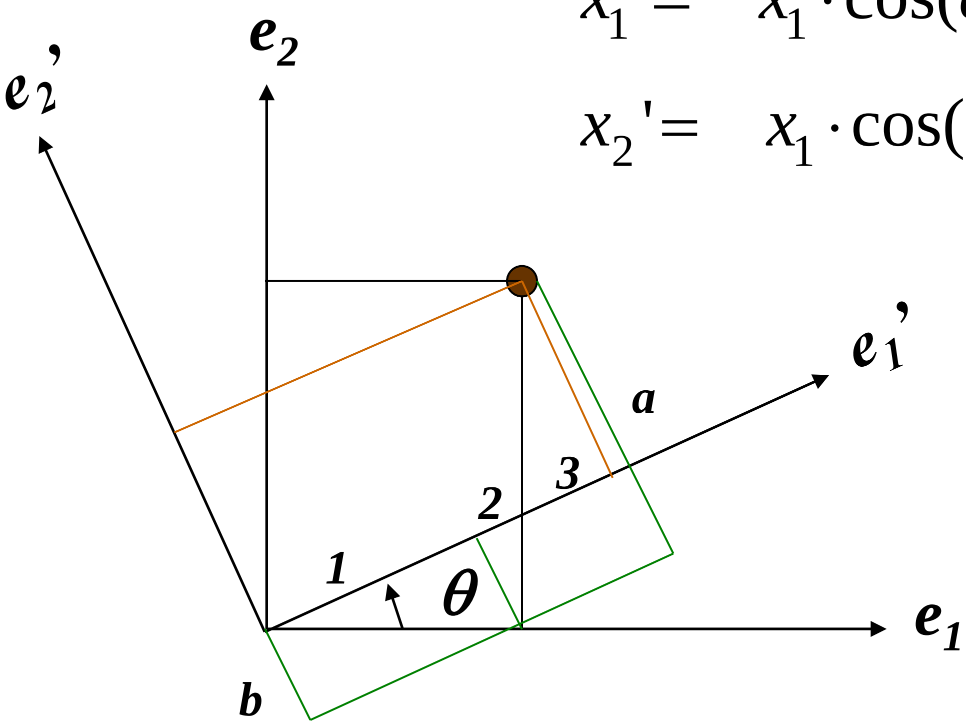
座標轉換

$$x_1' = x_1 \cdot \cos\theta + x_2 \sin\theta$$
$$x_2' = -x_1 \cdot \sin\theta + x_2 \cos\theta$$

$$x_i' = \beta_{ij} \cdot x_j$$

$$x_1' = x_1 \cdot \cos(e_1', e_1) + x_2 \cos(e_1', e_2)$$
$$x_2' = x_1 \cdot \cos(e_2', e_1) + x_2 \cos(e_2', e_2)$$



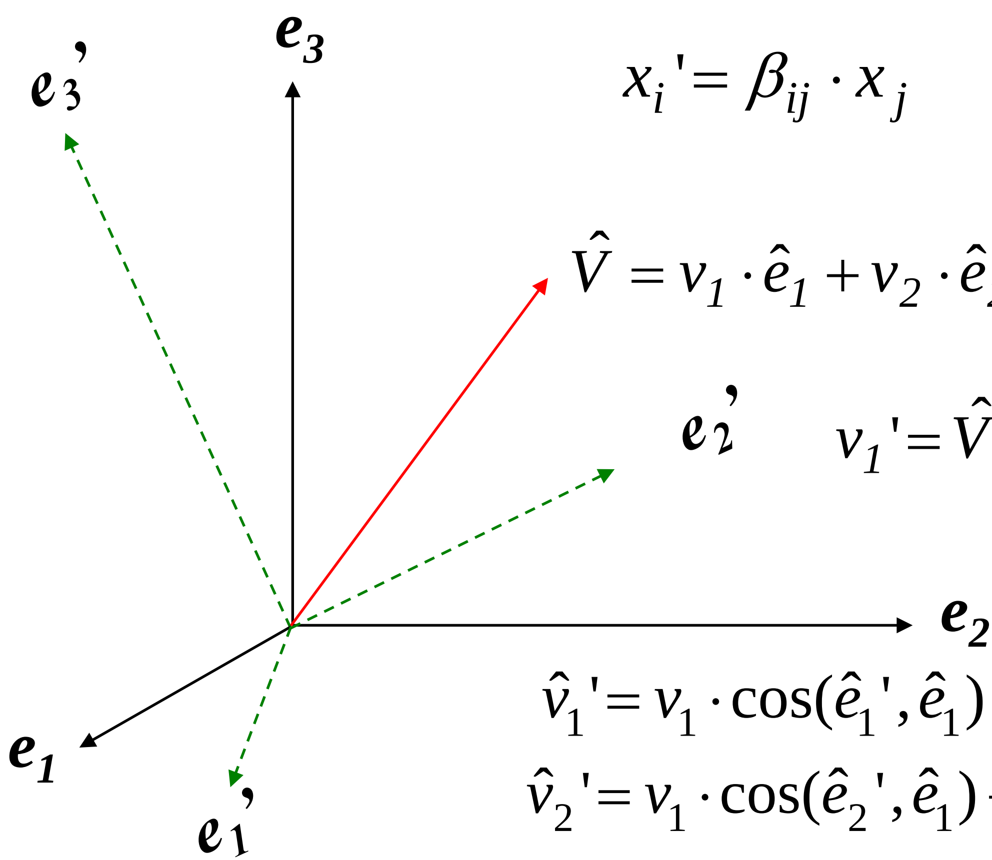
$$\beta_{ij} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$
$$= \begin{bmatrix} \cos(\hat{e}_1', \hat{e}_1) & \cos(\hat{e}_1', \hat{e}_2) \\ \cos(\hat{e}_2', \hat{e}_1) & \cos(\hat{e}_2', \hat{e}_2) \end{bmatrix}$$

座標轉換

$$x_1' = x_1 \cdot \cos(e_1', e_1) + x_2 \cos(e_1', e_2)$$

$$x_2' = x_1 \cdot \cos(e_2', e_1) + x_2 \cos(e_2', e_2)$$

$$x_i' = \beta_{ij} \cdot x_j \qquad \beta_{ij} = \begin{bmatrix} \cos(\hat{e}_1', \hat{e}_1) & \cos(\hat{e}_1', \hat{e}_2) \\ \cos(\hat{e}_2', \hat{e}_1) & \cos(\hat{e}_2', \hat{e}_2) \end{bmatrix}$$



$$\hat{V} = v_1 \cdot \hat{e}_1 + v_2 \cdot \hat{e}_2 + v_3 \cdot \hat{e}_3 = v_1' \cdot \hat{e}_1' + v_2' \cdot \hat{e}_2' + v_3' \cdot \hat{e}_3'$$

$$v_1' = \hat{V} \cdot \hat{e}_1'; \quad v_2' = \hat{V} \cdot \hat{e}_2'; \quad v_3' = \hat{V} \cdot \hat{e}_3'$$

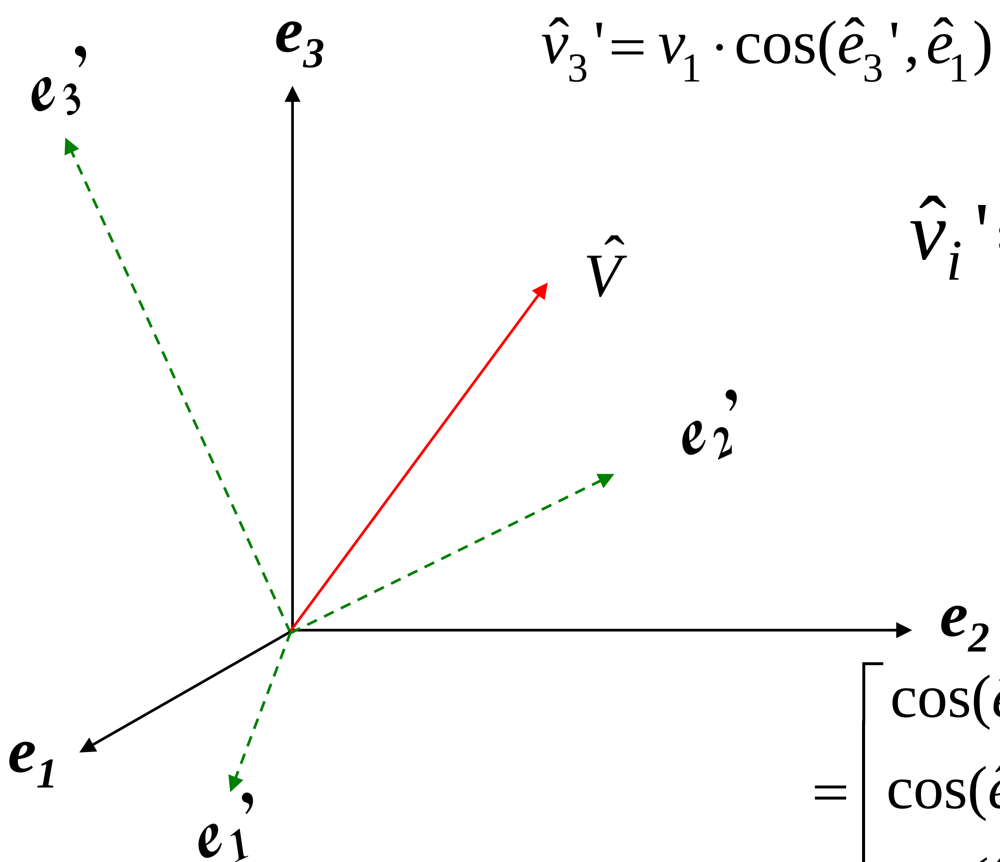
$$\hat{v}_1' = v_1 \cdot \cos(\hat{e}_1', \hat{e}_1) + v_2 \cdot \cos(\hat{e}_1', \hat{e}_2) + v_3 \cdot \cos(\hat{e}_1', \hat{e}_3)$$

$$\hat{v}_2' = v_1 \cdot \cos(\hat{e}_2', \hat{e}_1) + v_2 \cdot \cos(\hat{e}_2', \hat{e}_2) + v_3 \cdot \cos(\hat{e}_2', \hat{e}_3)$$

$$\hat{v}_3' = v_1 \cdot \cos(\hat{e}_3', \hat{e}_1) + v_2 \cdot \cos(\hat{e}_3', \hat{e}_2) + v_3 \cdot \cos(\hat{e}_3', \hat{e}_3)$$

座標轉換

$$\hat{v}_1' = v_1 \cdot \cos(\hat{e}_1', \hat{e}_1) + v_2 \cdot \cos(\hat{e}_1', \hat{e}_2) + v_3 \cdot \cos(\hat{e}_1', \hat{e}_3)$$
$$\hat{v}_2' = v_1 \cdot \cos(\hat{e}_2', \hat{e}_1) + v_2 \cdot \cos(\hat{e}_2', \hat{e}_2) + v_3 \cdot \cos(\hat{e}_2', \hat{e}_3)$$
$$\hat{v}_3' = v_1 \cdot \cos(\hat{e}_3', \hat{e}_1) + v_2 \cdot \cos(\hat{e}_3', \hat{e}_1) + v_3 \cdot \cos(\hat{e}_3', \hat{e}_3)$$



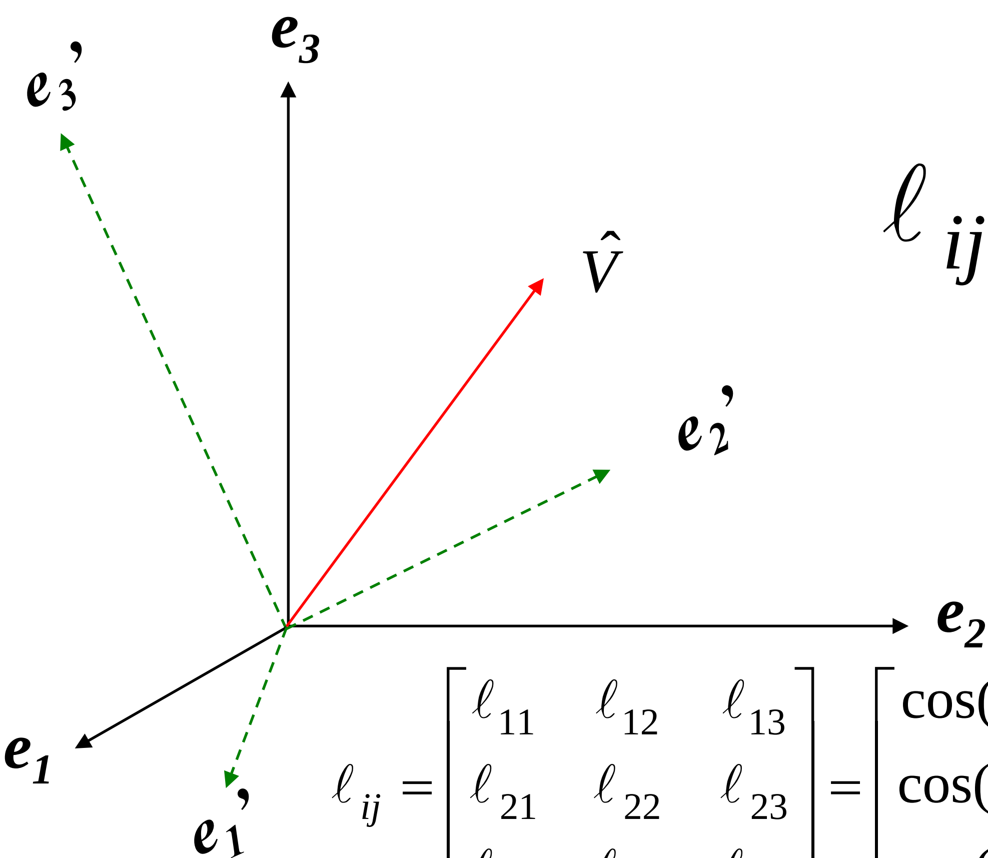
$$\hat{v}_i' = \ell_{ij} \cdot v_j \qquad \ell_{ij} = \hat{e}_i' \cdot \hat{e}_j$$

$$\ell_{ij} = \begin{bmatrix} \ell_{11} & \ell_{12} & \ell_{13} \\ \ell_{21} & \ell_{22} & \ell_{23} \\ \ell_{31} & \ell_{32} & \ell_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\hat{e}_1', \hat{e}_1) & \cos(\hat{e}_1', \hat{e}_2) & \cos(\hat{e}_1', \hat{e}_3) \\ \cos(\hat{e}_2', \hat{e}_1) & \cos(\hat{e}_2', \hat{e}_2) & \cos(\hat{e}_2', \hat{e}_3) \\ \cos(\hat{e}_3', \hat{e}_1) & \cos(\hat{e}_3', \hat{e}_2) & \cos(\hat{e}_3', \hat{e}_3) \end{bmatrix}$$

座標轉換

$$\hat{v}_i' = \ell_{ij} \cdot v_j$$



$$\ell_{ij} = \hat{e}_i' \cdot \hat{e}_j$$

方向餘弦

$$\ell_{ij} = \begin{bmatrix} \ell_{11} & \ell_{12} & \ell_{13} \\ \ell_{21} & \ell_{22} & \ell_{23} \\ \ell_{31} & \ell_{32} & \ell_{33} \end{bmatrix} = \begin{bmatrix} \cos(\hat{e}_1', \hat{e}_1) & \cos(\hat{e}_1', \hat{e}_2) & \cos(\hat{e}_1', \hat{e}_3) \\ \cos(\hat{e}_2', \hat{e}_1) & \cos(\hat{e}_2', \hat{e}_2) & \cos(\hat{e}_2', \hat{e}_3) \\ \cos(\hat{e}_3', \hat{e}_1) & \cos(\hat{e}_3', \hat{e}_2) & \cos(\hat{e}_3', \hat{e}_3) \end{bmatrix}$$

HW2

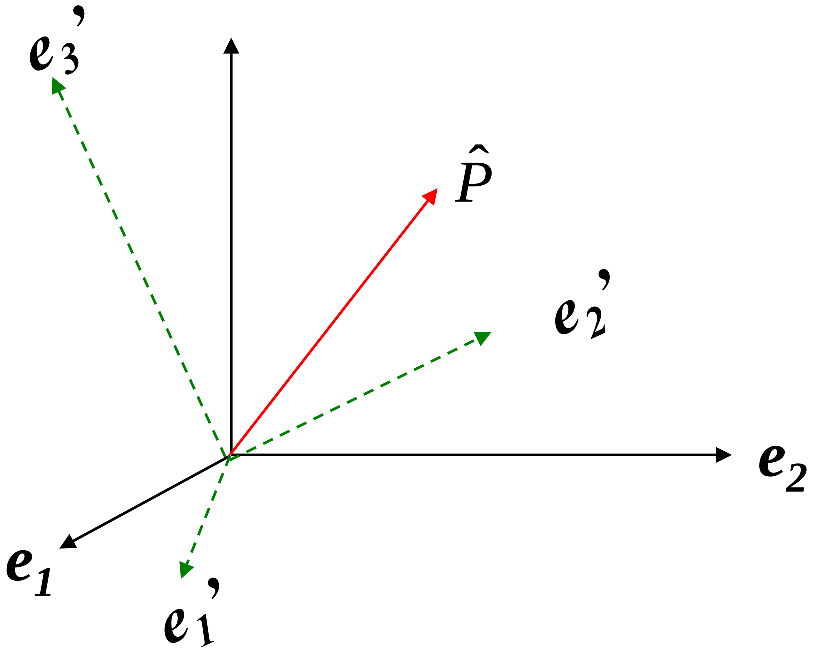
某一位移向量 $\hat{P}$ *於原座標系統為* $\hat{P} = u \cdot \hat{e}_1 + v \cdot \hat{e}_2 + w \cdot \hat{e}_3 (u = 3m; v = 3m; w = 3m)$

請問於旋轉後新座標系統此一位移向量 $\hat{P}'$ *為何？*

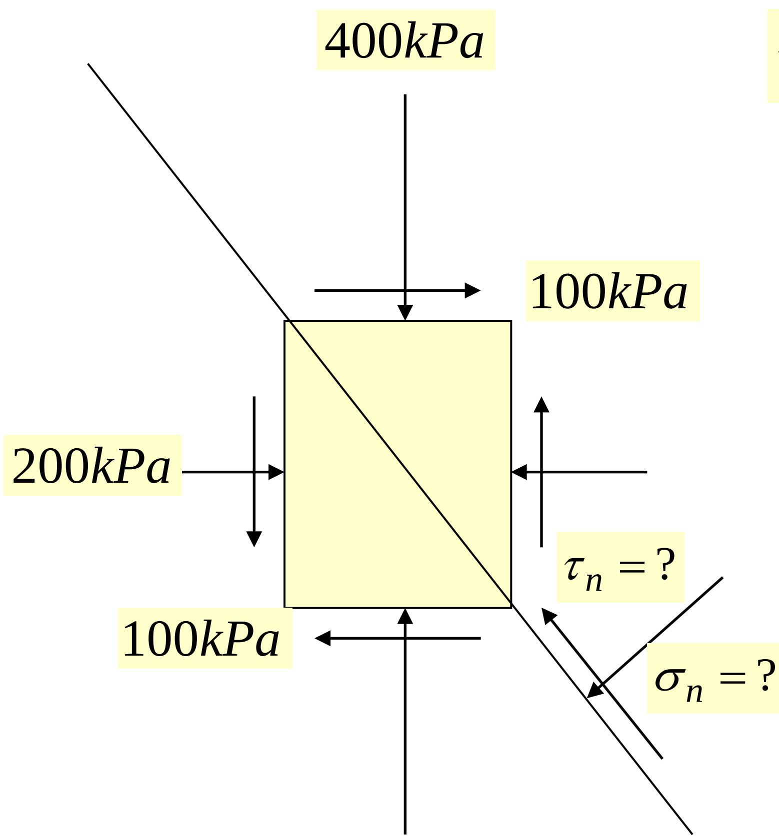
$$\hat{e}_1' = 0.071 \cdot \hat{e}_1 + 0.816 \cdot \hat{e}_2 + 0.574 \cdot \hat{e}_3$$

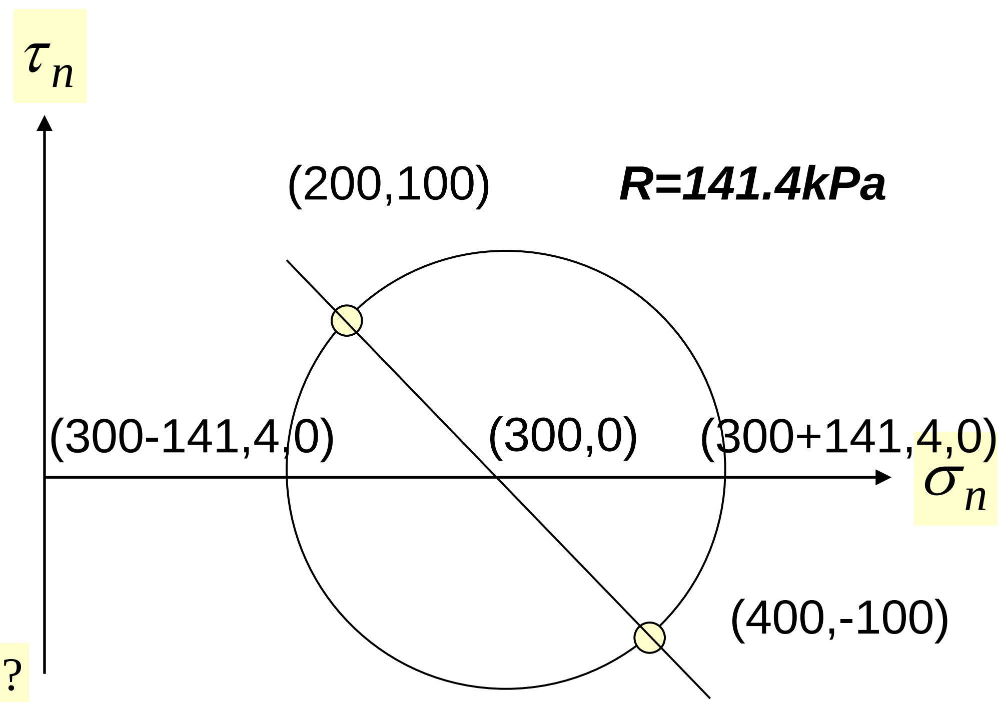
$$\hat{e}_2' = -0.584 \cdot \hat{e}_1 - 0.440 \cdot \hat{e}_2 + 0.682 \cdot \hat{e}_3$$

$$\hat{e}_3' = 0.808 \cdot \hat{e}_1 - 0.377 \cdot \hat{e}_2 + 0.454 \cdot \hat{e}_3$$



Construct a Mohr circle



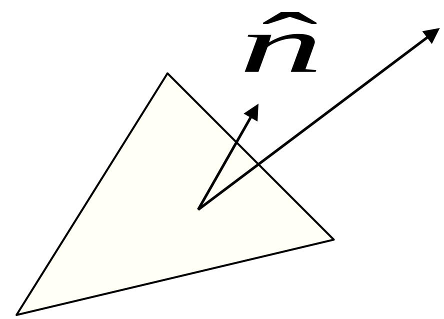


100kPa

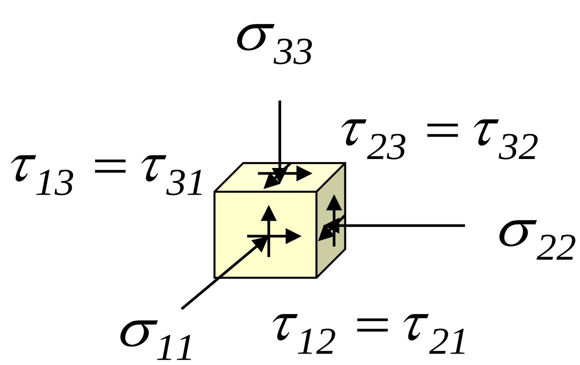
圓心？
半徑？
最大主應力？
最小主應力？
主應力方向？

主應力之定義為何?

Principal stress and principal direction



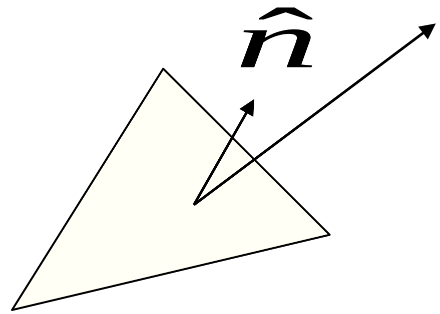
$$\overset{\hat{n}}{T} = \overset{1}{T}\cdot n_1 + \overset{2}{T}\cdot n_2 + \overset{3}{T}\cdot n_3$$



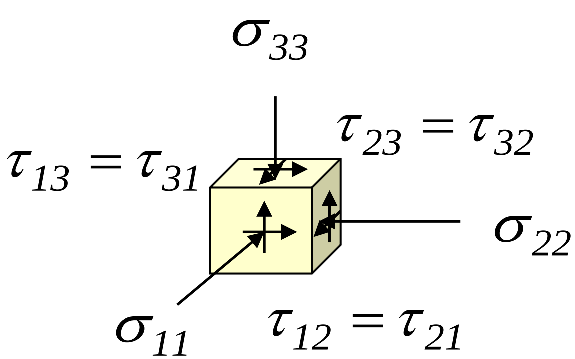
$$\overset{\hat{n}}{T}_i = \sigma_{ji}\cdot n_j = \sigma_{ij}\cdot n_j$$

剪應力為0之平面為主應力面
該面上作用之正向應力為主應力

Principal stress and principal direction



$$\overset{\hat{n}}{T} = \overset{1}{T} \cdot n_1 + \overset{2}{T} \cdot n_2 + \overset{3}{T} \cdot n_3$$



$$\overset{\hat{n}}{T}_i = \hat{\sigma}_n = \sigma \cdot n_i \qquad \tau_n = 0$$

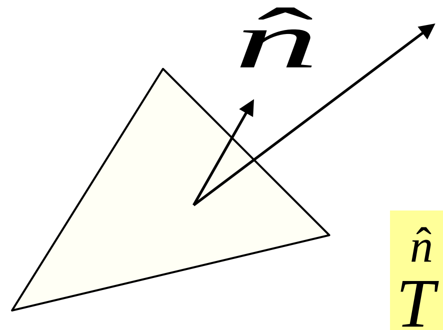
Principal stress $\hat{\sigma}_n$

principal direction $\hat{n}$

剪應力為0之平面為主應力面
該面上作用之正向應力為主應力

Three dimensional stress analysis

Principal stress and principal direction

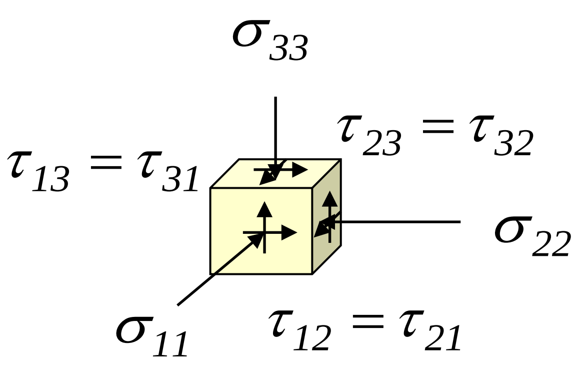


$$\overset{\hat{n}}{T} = \overset{1}{T}\cdot n_1 + \overset{2}{T}\cdot n_2 + \overset{3}{T}\cdot n_3$$

$$\overset{\hat{n}}{T}_i = \hat{\sigma}_n = \sigma \cdot n_i \qquad \tau_n = 0$$

$$\overset{\hat{n}}{T}_i = \sigma \cdot n_i \qquad \overset{\hat{n}}{T}_i = \sigma_{ij} \cdot n_j$$

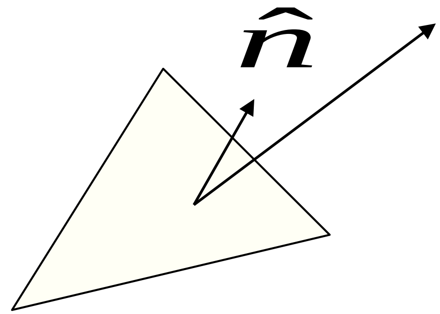
$$\sigma_{ij} \cdot n_j = \sigma \cdot n_i$$



剪應力為0之平面為主應力面
該面上作用之正向應力為主應力

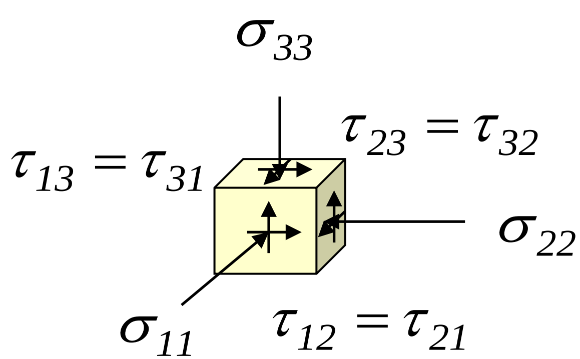
Three dimensional stress analysis

Principal stress and principal direction



$$\overset{\hat{n}}{T} = \overset{1}{T} \cdot n_1 + \overset{2}{T} \cdot n_2 + \overset{3}{T} \cdot n_3$$

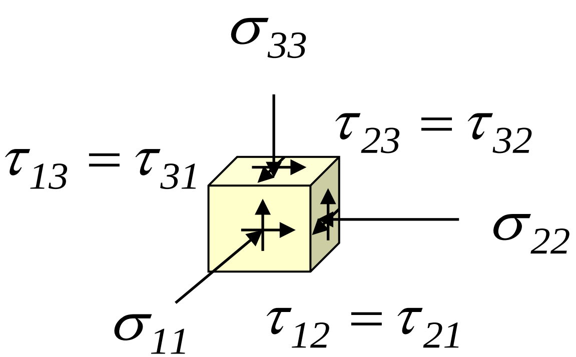
$$\sigma_{ij} \cdot n_j = \sigma \cdot n_i$$

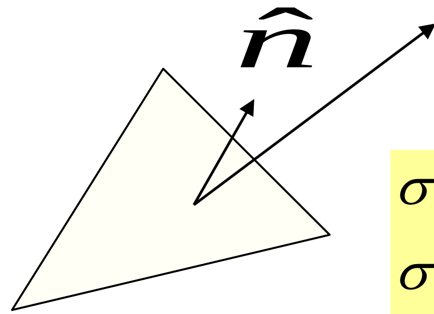


$$\sigma_{11} \cdot n_1 + \sigma_{12} \cdot n_2 + \sigma_{13} \cdot n_3 = \sigma \cdot n_1$$
$$\sigma_{21} \cdot n_1 + \sigma_{22} \cdot n_2 + \sigma_{23} \cdot n_3 = \sigma \cdot n_2$$
$$\sigma_{31} \cdot n_1 + \sigma_{32} \cdot n_2 + \sigma_{33} \cdot n_3 = \sigma \cdot n_3$$

剪應力為0之平面為主應力面
該面上作用之正向應力為主應力

Principal stress and principal direction





$$\overset{\hat{n}}{T} = \overset{1}{T} \cdot n_1 + \overset{2}{T} \cdot n_2 + \overset{3}{T} \cdot n_3$$

$$\sigma_{11} \cdot n_1 + \sigma_{12} \cdot n_2 + \sigma_{13} \cdot n_3 = \sigma \cdot n_1$$
$$\sigma_{21} \cdot n_1 + \sigma_{22} \cdot n_2 + \sigma_{23} \cdot n_3 = \sigma \cdot n_2$$
$$\sigma_{31} \cdot n_1 + \sigma_{32} \cdot n_2 + \sigma_{33} \cdot n_3 = \sigma \cdot n_3$$

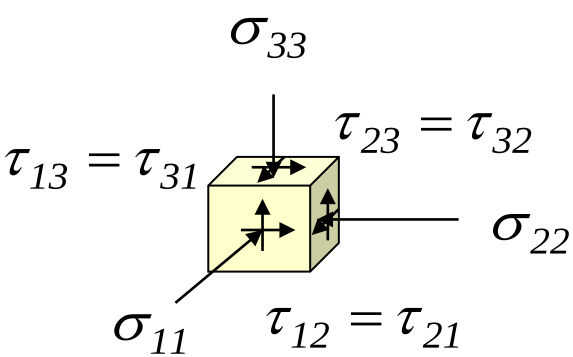
剪應力為0之平面為主應力面
該面上作用之正向應力為主應力

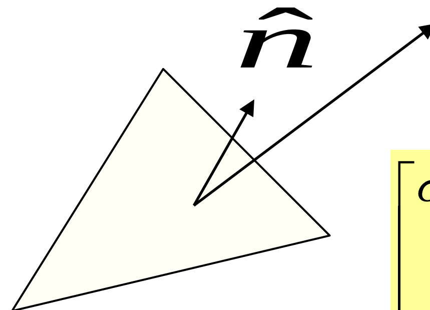
$$\begin{bmatrix} \sigma_{11} - \sigma & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} - \sigma & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} - \sigma \end{bmatrix} \cdot \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

For non-trivial solution of n

$$\begin{vmatrix} \sigma_{11} - \sigma & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} - \sigma & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} - \sigma \end{vmatrix} = 0$$

Principal stress and principal direction





$$\overset{\hat{n}}{T} = \overset{1}{T} \cdot n_1 + \overset{2}{T} \cdot n_2 + \overset{3}{T} \cdot n_3$$

$$\begin{bmatrix} \sigma_{11}-\sigma & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22}-\sigma & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33}-\sigma \end{bmatrix} \cdot \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

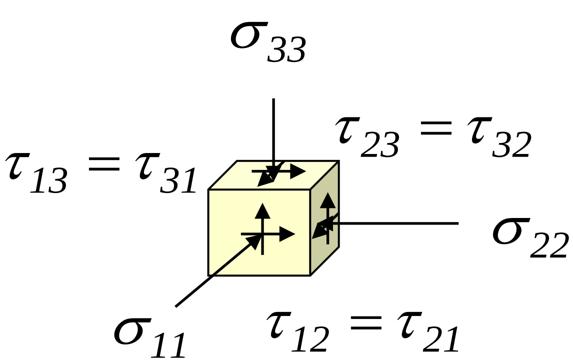
剪應力為 0 之平面為主應力面
該面上作用之正向應力為主應力

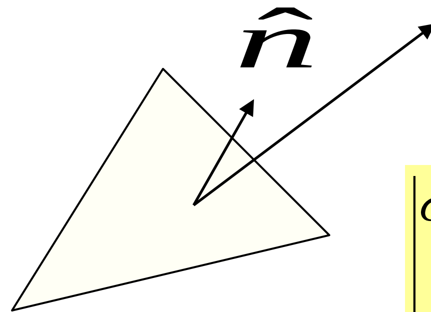
$$\begin{vmatrix} \sigma_{11}-\sigma & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22}-\sigma & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33}-\sigma \end{vmatrix} = 0$$

3 solutions of stress represent 3 principal stresses
Correspond orthogonal directions are principal directions

Three dimensional stress analysis

Principal stress and principal direction





$$\overset{\hat{n}}{T} = \overset{1}{T}\cdot n_1 + \overset{2}{T}\cdot n_2 + \overset{3}{T}\cdot n_3$$

$$\begin{vmatrix} \sigma_{11}-\sigma & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22}-\sigma & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33}-\sigma \end{vmatrix} = 0$$

剪應力為0之平面為主應力面
該面上作用之正向應力為主應力

$$\sigma^3 - I_1\cdot\sigma^2 + I_2\cdot\sigma - I_3 = 0$$

3 solutions of stress represent 3 principal stresses $\sigma_1, \sigma_2, \sigma_3$

Correspond orthogonal directions are principal directions

$$\begin{bmatrix} \sigma_{11}-\sigma_1 & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22}-\sigma_1 & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33}-\sigma_1 \end{bmatrix} \cdot \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

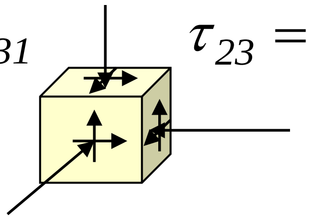
$\sigma = \sigma_1$ $\qquad$ $n_1^{(1)}, n_2^{(1)}, n_3^{(1)}$

Three dimensional stress analysis

$\sigma_{33} = 1$

$\tau_{13} = \tau_{31} = 0$

$\tau_{23} = \tau_{32} = 0$



$\sigma_{22} = 0$

$\sigma_{11} = 1$

$\tau_{12} = \tau_{21} = \sqrt{3}$

$$\sigma_{ij} = \begin{bmatrix} 1 & \sqrt{3} & 0 \\ \sqrt{3} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Principal stresses =?
Principal directions =?

$$\begin{vmatrix} \sigma_{11} - \sigma & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} - \sigma & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} - \sigma \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 - \sigma & \sqrt{3} & 0 \\ \sqrt{3} & -\sigma & 0 \\ 0 & 0 & 1 - \sigma \end{vmatrix} = 0$$

$$(\sigma - 1)(\sigma^2 - \sigma - 3) = 0$$

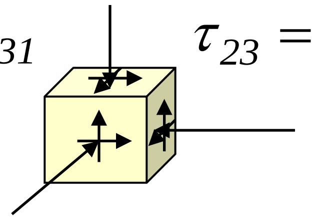
3 solutions of stress represent 3 principal stresses

$$\sigma_1 = \frac{1 + \sqrt{13}}{2}, \sigma_2 = 1, \sigma_3 = \frac{1 - \sqrt{13}}{2}$$

Three dimensional stress analysis

$\sigma_{33} = 1$

$\tau_{13} = \tau_{31} = 0$ $\quad \tau_{23} = \tau_{32} = 0$



$\sigma_{22} = 0$

$\sigma_{11} = 1$ $\quad \tau_{12} = \tau_{21} = \sqrt{3}$

$$\sigma_{ij} = \begin{bmatrix} 1 & \sqrt{3} & 0 \\ \sqrt{3} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Principal stresses =?
Principal directions =?

$$\begin{bmatrix} \sigma_{11} - \sigma_2 & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} - \sigma_2 & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} - \sigma_2 \end{bmatrix} \cdot \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$(\sigma - 1)(\sigma^2 - \sigma - 3) = 0$$

$$\sigma_1 = \frac{1+\sqrt{13}}{2}, \sigma_2 = 1, \sigma_3 = \frac{1-\sqrt{13}}{2}$$

$$\begin{bmatrix} 1-1 & \sqrt{3} & 0 \\ \sqrt{3} & 0-1 & 0 \\ 0 & 0 & 1-1 \end{bmatrix} \cdot \begin{bmatrix} n_1^{(2)} \\ n_2^{(2)} \\ n_3^{(2)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sigma = \sigma_2$$

Correspond orthogonal directions are principal directions

$$(n_1^{(2)}, n_2^{(2)}, n_3^{(2)}) = (0,0,1)$$

$$n_1^{\ 2} + n_2^{\ 2} + n_3^{\ 2} = 1$$

$$(n_1^{(1)}, n_2^{(1)}, n_3^{(1)}) = (0.8,0.6,0); (n_1^{(3)}, n_2^{(3)}, n_3^{(3)}) = (0.6,-0.8,0)$$

HW3

Given a stress tensor $\sigma_{ij} = \begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$ (GPa)，

Please find：(1) three principal stresses；(2) three principal directions。

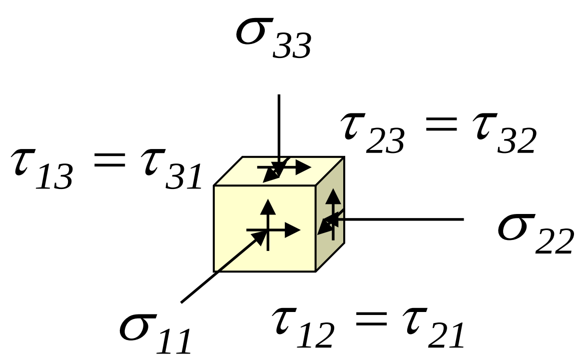
Given a fault plane N-S strike、dip angle 90° (vertical plane)

Please find：(1) Surface traction $\overset{\hat{n}}{T}_i$ ；(2)normal stress；(3)shear stress。

If the stress state of the fault was measured just before the fault slip (shear stress=shear strength), please determine the friction coefficient of the fault ?

Hint: direction 1 in Eastern-ward, direction 2 in Northern-ward, direction 3 in up-ward, friction coefficient $\mu = \tau_n / \sigma_n$

The invariants of stress tensor



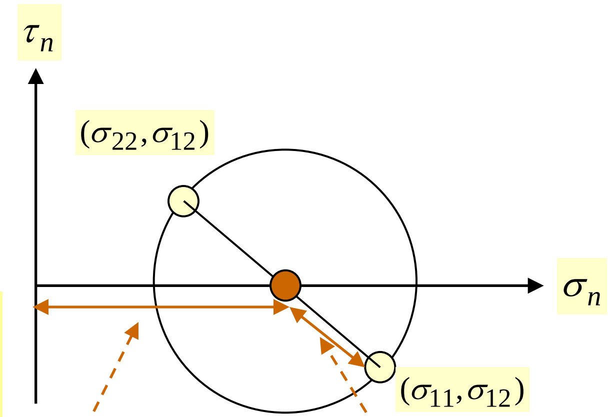
$$\begin{vmatrix} \sigma_{11}-\sigma & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22}-\sigma & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33}-\sigma \end{vmatrix} = 0$$

$$\sigma^3 - I_1 \cdot \sigma^2 + I_2 \cdot \sigma - I_3 = 0$$

$$I_1 = \sigma_{11} + \sigma_{22} + \sigma_{33}$$

$$I_2 = \begin{vmatrix} \sigma_{22} & \sigma_{23} \\ \sigma_{32} & \sigma_{33} \end{vmatrix} + \begin{vmatrix} \sigma_{11} & \sigma_{13} \\ \sigma_{31} & \sigma_{33} \end{vmatrix} + \begin{vmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{vmatrix}$$

$$I_3 = \begin{vmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{vmatrix}$$



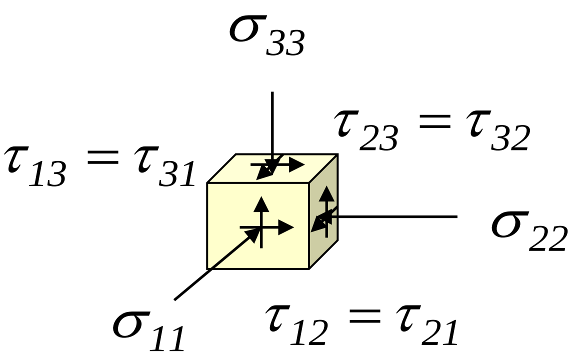
For two dimensional

$$\sigma^2 - (\sigma_{11} + \sigma_{22}) \cdot \sigma + (\sigma_{11} \cdot \sigma_{22} - \sigma_{12}^{\ 2}) = 0$$

$$2 \times (\frac{\sigma_{11}+\sigma_{22}}{2}) \qquad (\frac{\sigma_{11}+\sigma_{22}}{2})^2 - [(\frac{\sigma_{11}-\sigma_{22}}{2})^2 + \sigma_{12}^{\ 2}]$$

The invariants of stress tensor

$$\sigma^3 - I_1 \cdot \sigma^2 + I_2 \cdot \sigma - I_3 = 0$$



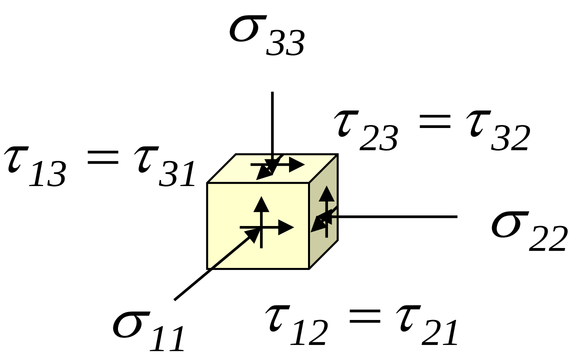
$$\sigma_{ij} = \begin{bmatrix} 1 & \sqrt{3} & 0 \\ \sqrt{3} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \sigma_{ij} = \begin{bmatrix} \frac{1+\sqrt{13}}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1-\sqrt{13}}{2} \end{bmatrix}$$

$$I_1 = \sigma_{11} + \sigma_{22} + \sigma_{33} = 2$$

$$I_2 = \begin{vmatrix} \sigma_{22} & \sigma_{23} \\ \sigma_{32} & \sigma_{33} \end{vmatrix} + \begin{vmatrix} \sigma_{11} & \sigma_{13} \\ \sigma_{31} & \sigma_{33} \end{vmatrix} + \begin{vmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{vmatrix} = -2$$

$$I_3 = \begin{vmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{vmatrix} = -3$$

The deviator stress



$$\sigma_{ij}^{d} = \sigma_{ij} - \frac{I_1}{3}\delta_{ij}$$

$$\sigma_{ij}^{d} = \begin{bmatrix} \sigma_{xx} - \sigma_m & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} - \sigma_m & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} - \sigma_m \end{bmatrix}$$

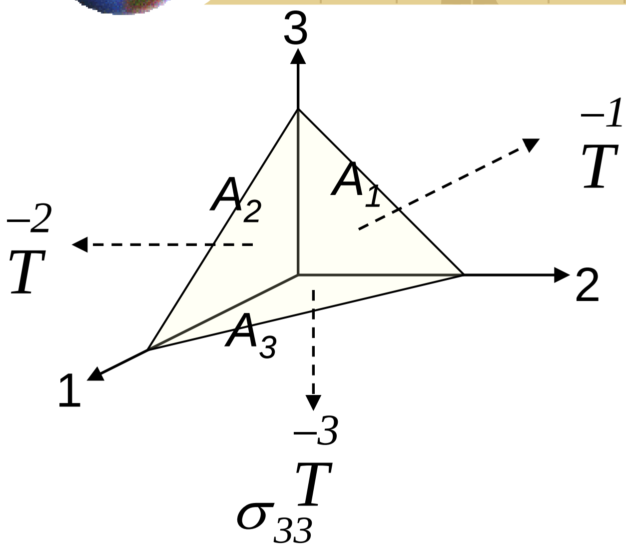
Principal deviator stresses

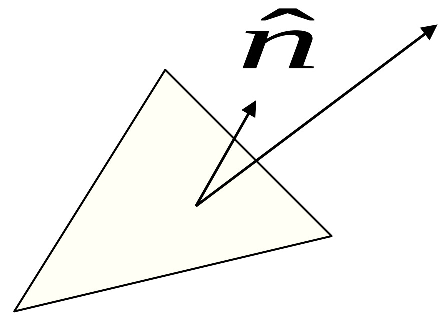
$$S_1 = \sigma_1 - \frac{I_1}{3},$$

$$S_2 = \sigma_2 - \frac{I_1}{3},$$

$$S_3 = \sigma_3 - \frac{I_1}{3},$$

Stress transformation



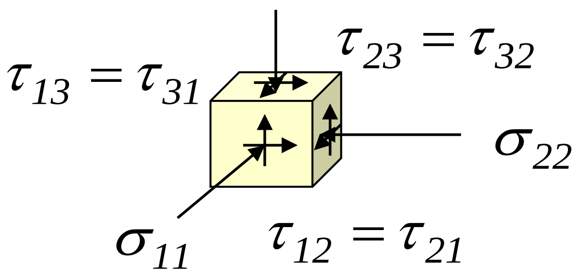


$$\overset{\hat{n}}{T} = \overset{1}{T}\cdot n_1 + \overset{2}{T}\cdot n_2 + \overset{3}{T}\cdot n_3$$

$$\overset{\hat{n}}{T}_i = \sigma_{ji}\cdot n_j = \sigma_{ij}\cdot n_j$$

In matrix form

$$\begin{bmatrix} \overset{\hat{n}}{T}_1 \\ \overset{\hat{n}}{T}_2 \\ \overset{\hat{n}}{T}_3 \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \cdot \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$



$$n_l' = \ell_{lj}\cdot n_j$$

$$\ell_{lk}\cdot n_l' = \ell_{lk}\cdot \ell_{lj}\cdot n_j$$

$$= e_l'\cdot e_l'\cdot e_k\cdot e_j\cdot n_j$$

$$= 1\cdot \delta_{kj}\cdot n_j = n_k$$

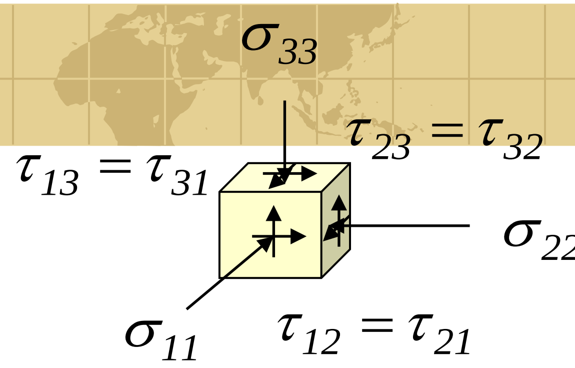
$$\overset{\hat{n}}{T}_i = \sigma_{ji}\cdot n_j = \sigma_{ij}\cdot n_j$$

$$\overset{\hat{n}}{T}_i' = \ell_{ij}\cdot \overset{\hat{n}}{T}_j = \ell_{ij}\cdot \sigma_{jk}\cdot n_k = \ell_{ij}\cdot \sigma_{jk}\cdot \ell_{lk}\cdot n_l'$$

$$\overset{\hat{n}}{T}_i' = \sigma_{il}'\cdot n_l'$$

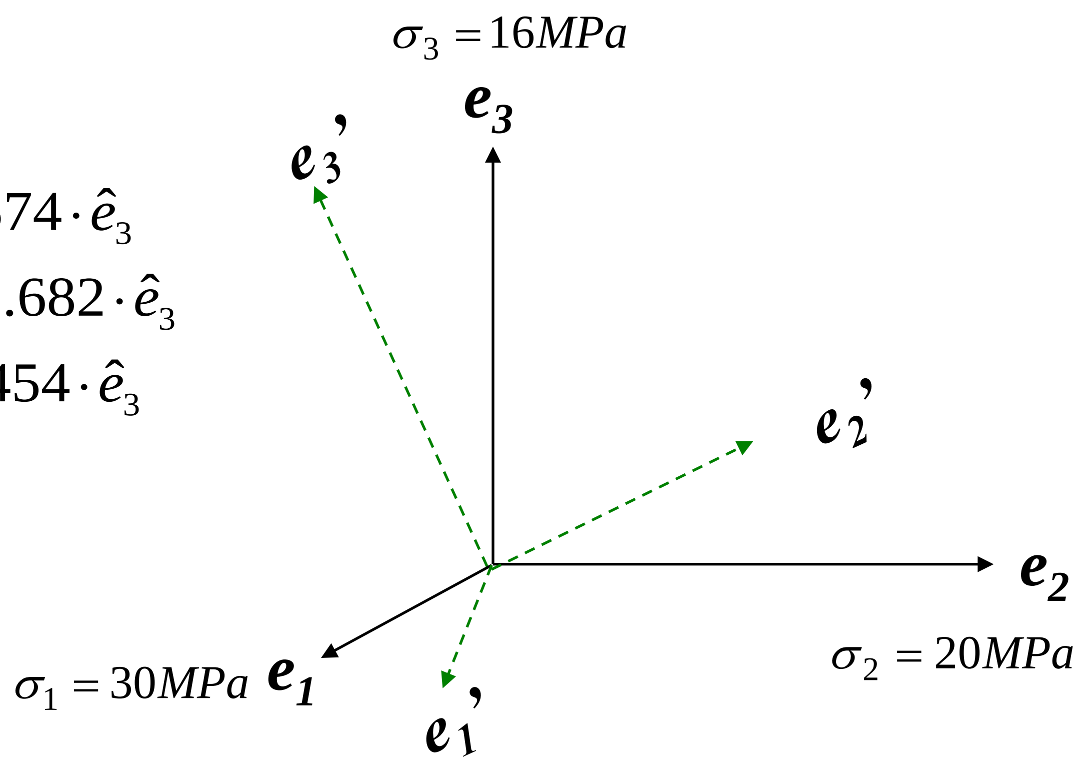
$$\therefore \sigma_{il}' = \ell_{ij}\cdot \sigma_{jk}\cdot \ell_{lk}$$

HW4



Please find the stress tensor in coordinate system e_1'-e_2'-e_3'

$$\hat{e}_1' = 0.071 \cdot \hat{e}_1 + 0.816 \cdot \hat{e}_2 + 0.574 \cdot \hat{e}_3$$
$$\hat{e}_2' = -0.584 \cdot \hat{e}_1 - 0.440 \cdot \hat{e}_2 + 0.682 \cdot \hat{e}_3$$
$$\hat{e}_3' = 0.808 \cdot \hat{e}_1 - 0.377 \cdot \hat{e}_2 + 0.454 \cdot \hat{e}_3$$



Please read section 2.3

大域座標下 $\sigma_{pq} = ?$

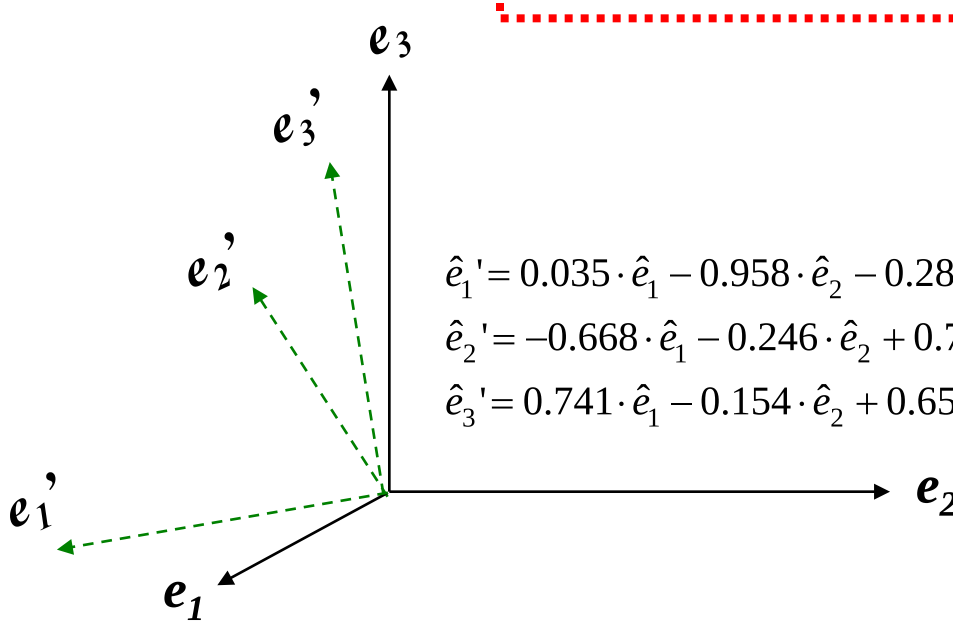
$$\ell_{ij} = \hat{e}_i{}' \cdot \hat{e}_j$$

$$\sigma_{il}{}' = \ell_{ij} \cdot \sigma_{jk} \cdot \ell_{lk}$$

$$\therefore \ell_{ip} \cdot \sigma_{il}{}' \cdot \ell_{lq} = \ell_{ip} \cdot \ell_{ij} \cdot \sigma_{jk} \cdot \ell_{lk} \cdot \ell_{lq} = \delta_{pj} \cdot \sigma_{jk} \cdot \delta_{kq} = \sigma_{pq}$$

$$\therefore \sigma_{pq} = \ell_{ip} \cdot \sigma_{il}{}' \cdot \ell_{lq} \qquad \textbf{or} \qquad \sigma_{il} = \ell_{ji} \cdot \sigma_{jk}{}' \cdot \ell_{kl}$$

$$\sigma_{jk}{}' = \begin{bmatrix} 36.6 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 12.3 \end{bmatrix} (MPa)$$



$$\hat{e}_1{}' = 0.035 \cdot \hat{e}_1 - 0.958 \cdot \hat{e}_2 - 0.284 \cdot \hat{e}_3$$
$$\hat{e}_2{}' = -0.668 \cdot \hat{e}_1 - 0.246 \cdot \hat{e}_2 + 0.702 \cdot \hat{e}_3$$
$$\hat{e}_3{}' = 0.741 \cdot \hat{e}_1 - 0.154 \cdot \hat{e}_2 + 0.653 \cdot \hat{e}_3$$

Stress transformation

$$\overset{\hat{n}}{T}_i = \sigma_{ji} \cdot n_j = \sigma_{ij} \cdot n_j$$

$$\overset{\hat{n}}{T}_i' = \ell_{ij} \cdot \overset{\hat{n}}{T}_j = \ell_{ij} \cdot \sigma_{jk} \cdot n_k$$

$$\overset{\hat{n}}{T}_i' = \sigma_{il}' \cdot n_l' = \sigma_{il}' \cdot \ell_{lk} \cdot n_k$$

$$\therefore \sigma_{il}' \cdot \ell_{lk} = \ell_{ij} \cdot \sigma_{jk}$$

$$n_l' = \ell_{lj} \cdot n_j$$

$$\ell_{lk} \cdot n_l' = \ell_{lk} \cdot \ell_{lj} \cdot n_j$$

$$= e_l' \cdot e_l' \cdot e_k \cdot e_j \cdot n_j$$

$$= 1 \cdot \delta_{kj} \cdot n_j = n_k$$

In matrix form

$$\begin{bmatrix} \sigma'_{11} & \sigma'_{12} & \sigma'_{13} \\ \sigma'_{21} & \sigma'_{22} & \sigma'_{23} \\ \sigma'_{31} & \sigma'_{32} & \sigma'_{33} \end{bmatrix} \cdot \begin{bmatrix} \ell_{11} & \ell_{12} & \ell_{13} \\ \ell_{21} & \ell_{22} & \ell_{23} \\ \ell_{31} & \ell_{32} & \ell_{33} \end{bmatrix} = \begin{bmatrix} \ell_{11} & \ell_{12} & \ell_{13} \\ \ell_{21} & \ell_{22} & \ell_{23} \\ \ell_{31} & \ell_{32} & \ell_{33} \end{bmatrix} \cdot \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} \sigma'_{11} & \sigma'_{12} & \sigma'_{13} \\ \sigma'_{21} & \sigma'_{22} & \sigma'_{23} \\ \sigma'_{31} & \sigma'_{32} & \sigma'_{33} \end{bmatrix} = \begin{bmatrix} \ell_{11} & \ell_{12} & \ell_{13} \\ \ell_{21} & \ell_{22} & \ell_{23} \\ \ell_{31} & \ell_{32} & \ell_{33} \end{bmatrix} \cdot \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \cdot \begin{bmatrix} \ell_{11} & \ell_{12} & \ell_{13} \\ \ell_{21} & \ell_{22} & \ell_{23} \\ \ell_{31} & \ell_{32} & \ell_{33} \end{bmatrix}^T \qquad \sigma_{il}' = \ell_{ij} \cdot \sigma_{jk} \cdot \ell_{lk}$$

$$\sigma_{il} = \ell_{ji} \cdot \sigma_{jk}' \cdot \ell_{kl} \qquad \textbf{OR} \qquad \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} = \begin{bmatrix} \ell_{11} & \ell_{12} & \ell_{13} \\ \ell_{21} & \ell_{22} & \ell_{23} \\ \ell_{31} & \ell_{32} & \ell_{33} \end{bmatrix}^T \cdot \begin{bmatrix} \sigma'_{11} & \sigma'_{12} & \sigma'_{13} \\ \sigma'_{21} & \sigma'_{22} & \sigma'_{23} \\ \sigma'_{31} & \sigma'_{32} & \sigma'_{33} \end{bmatrix} \cdot \begin{bmatrix} \ell_{11} & \ell_{12} & \ell_{13} \\ \ell_{21} & \ell_{22} & \ell_{23} \\ \ell_{31} & \ell_{32} & \ell_{33} \end{bmatrix}$$

Strength

- A stress state that the materials cannot sustain
- Failure