HW1

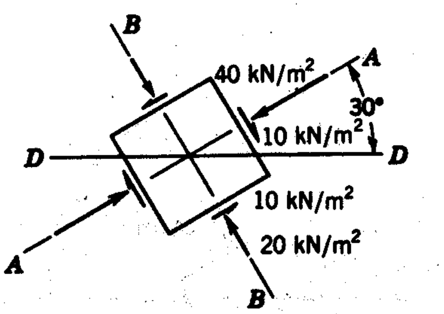
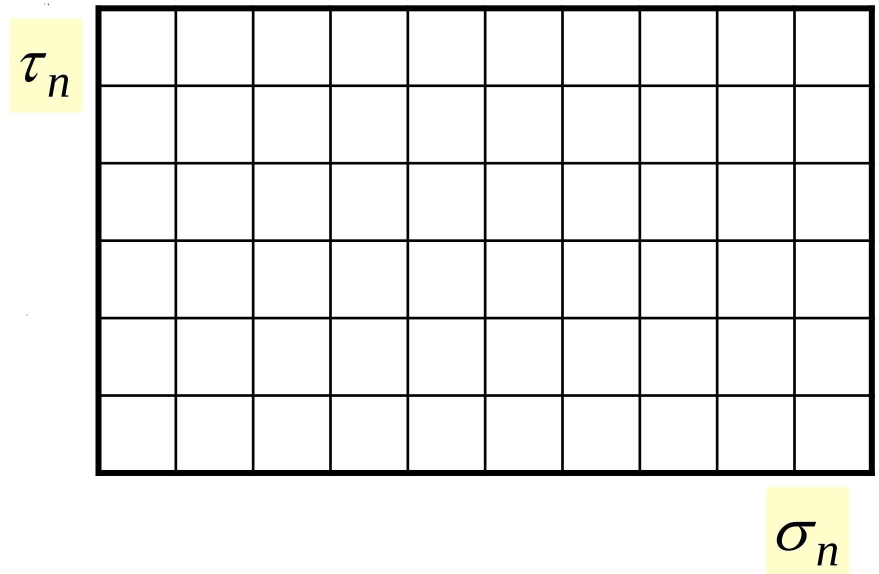


Fig. E8.6-1

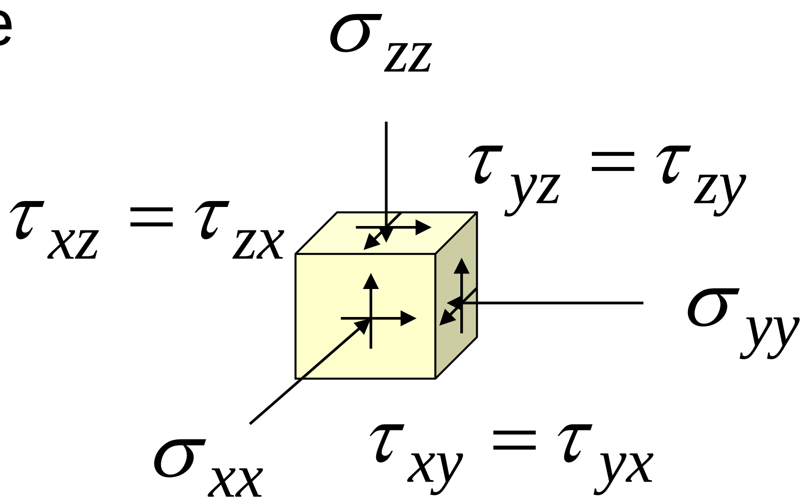


1. **Shear stress and Normal stress at horizontal plane**
2. **Two principal stresses and their orientations**

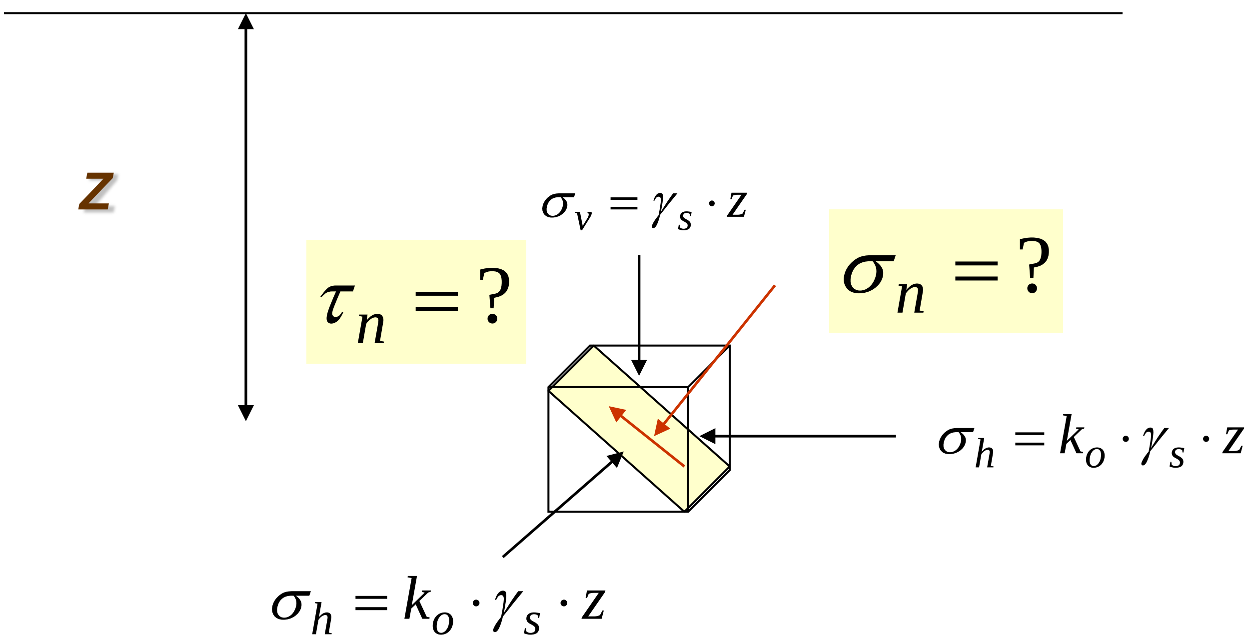
Three dimensional stress

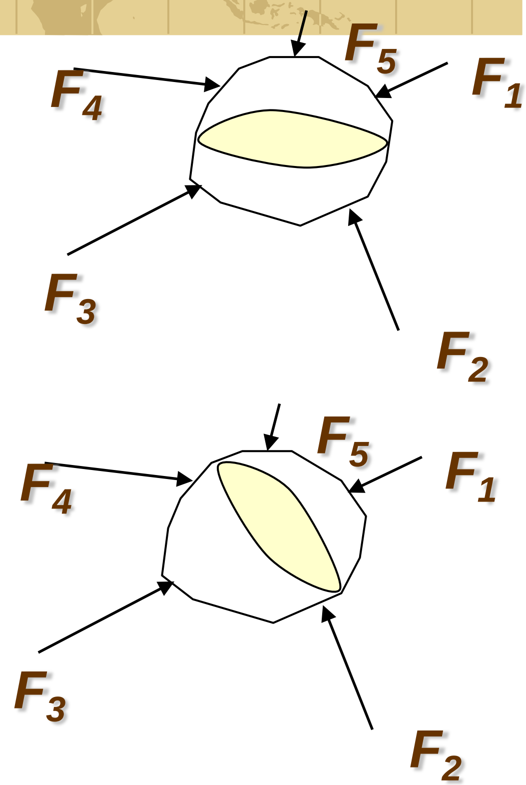
- Stress is a tensor
  - Defined by three surface tractions (vector) at three orthogonal plane

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$



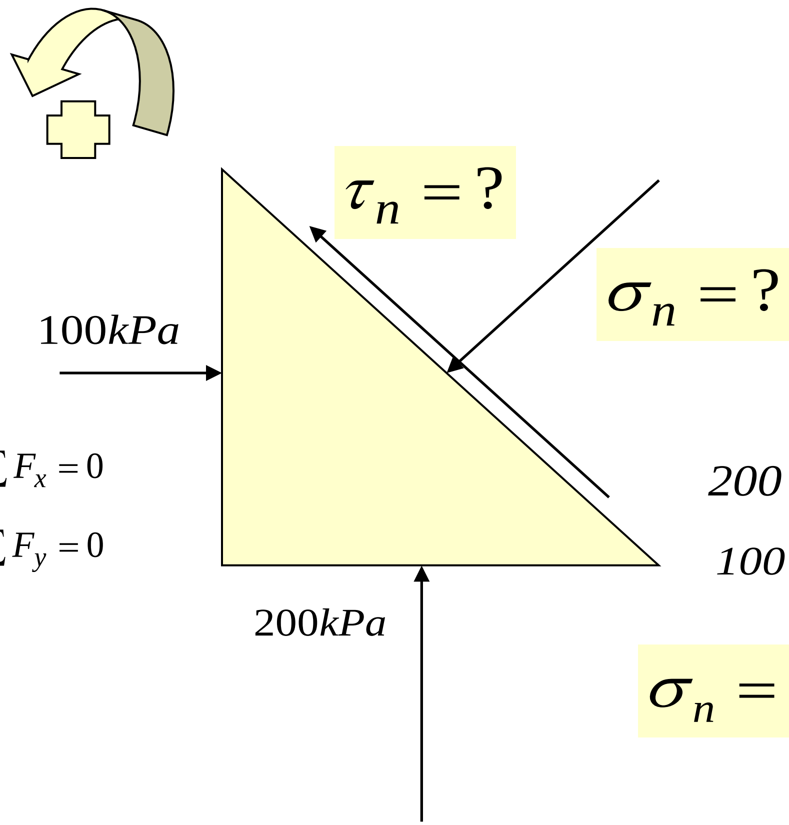
Normal stress and shear stress at an incline plane?





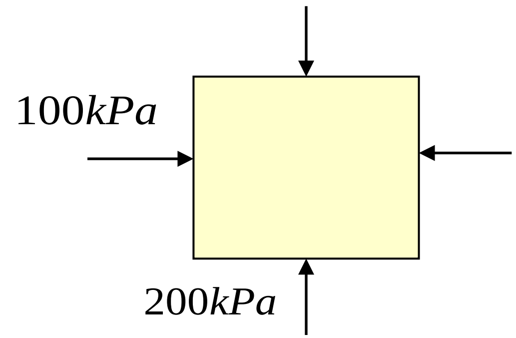
如　車籠埔斷層面上之正向應力與剪應力？
又如 某斷層面因蓄水庫造成水壓上升會不會被誘發活動？

Two dimensional stress



$\sum F_x = 0$

$\sum F_y = 0$



$$200 \times 1 = \sigma_n \times \sqrt{2} / \sqrt{2} - \tau_n \times \sqrt{2} / \sqrt{2}$$

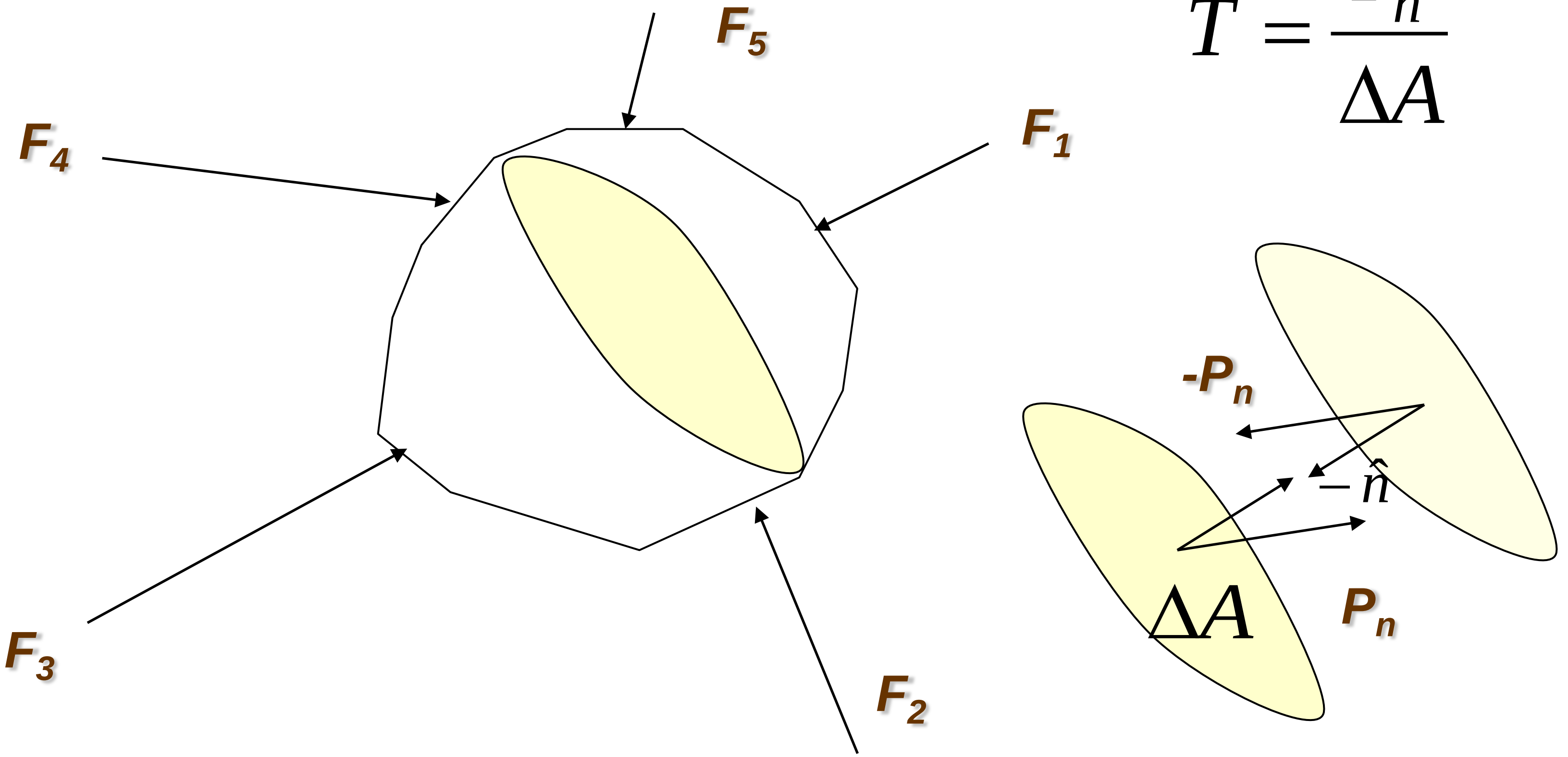
$$100 \times 1 = \sigma_n \times \sqrt{2} / \sqrt{2} + \tau_n \times \sqrt{2} / \sqrt{2}$$

$$\sigma_n = 150kPa \qquad \tau_n = -50kPa$$

How about three dimensional stress ?

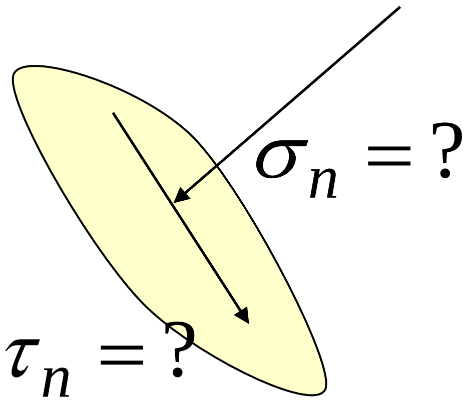
Surface traction $\overset{\hat{n}}{T}$

$$\overset{\hat{n}}{T} = \frac{P_n}{\Delta A}$$



Surface traction varied with the normal vector of concerned surface

Three dimensional stress analysis

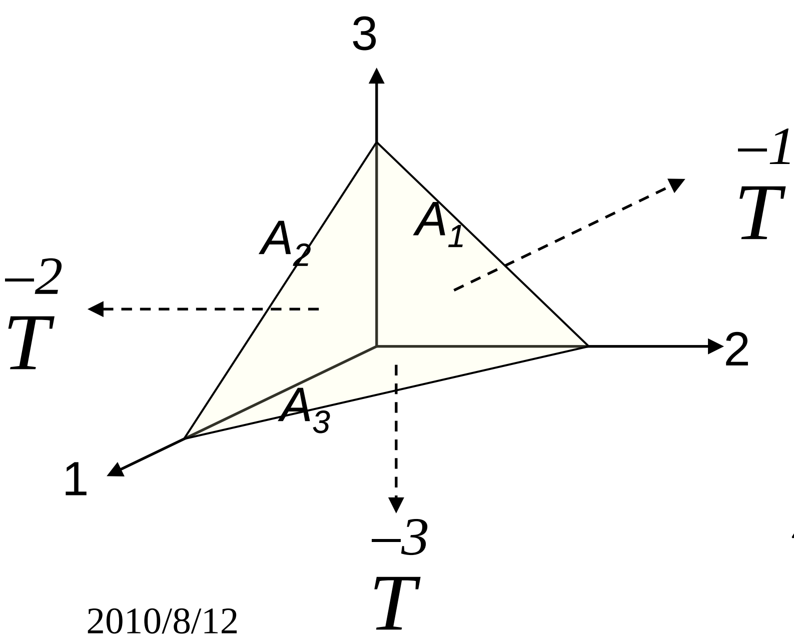


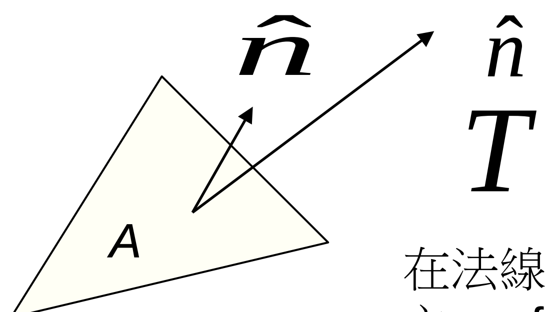
$$\overset{\hat{n}}{T}\cdot A + \overset{-1}{T}\cdot A_1 + \overset{-2}{T}\cdot A_2 + \overset{-3}{T}\cdot A_3 = 0$$

$$\overset{\hat{n}}{T}\cdot A = -(\overset{-1}{T}\cdot A_1 + \overset{-2}{T}\cdot A_2 + \overset{-3}{T}\cdot A_3)$$

$$\overset{\hat{n}}{T}\cdot A = \overset{1}{T}\cdot A_1 + \overset{2}{T}\cdot A_2 + \overset{3}{T}\cdot A_3$$

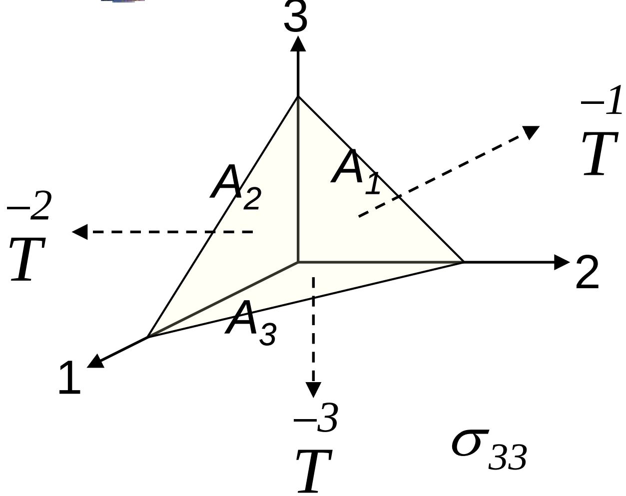
$$\overset{\hat{n}}{T} = \overset{1}{T}\cdot n_1 + \overset{2}{T}\cdot n_2 + \overset{3}{T}\cdot n_3$$

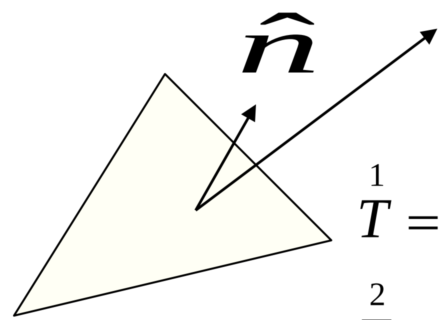




在法線向量為 $\hat{n}$ 之平面上之 surface traction

Three dimensional stress analysis



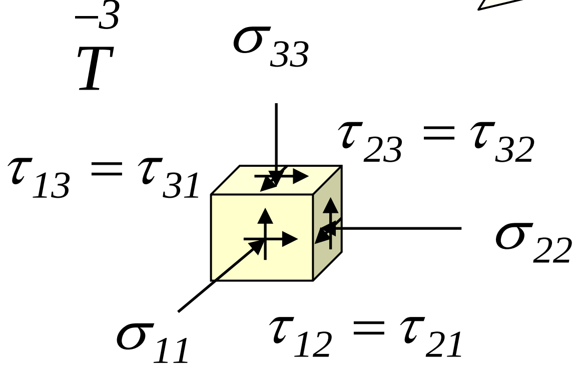


$$\overset{\hat{n}}{T} = \overset{1}{T}\cdot n_1 + \overset{2}{T}\cdot n_2 + \overset{3}{T}\cdot n_3$$

$$\overset{1}{T} = \sigma_{11}\cdot\hat{e}_1 + \sigma_{12}\cdot\hat{e}_2 + \sigma_{13}\cdot\hat{e}_3 = \sigma_{1i}\cdot\hat{e}_i$$

$$\overset{2}{T} = \sigma_{21}\cdot\hat{e}_1 + \sigma_{22}\cdot\hat{e}_2 + \sigma_{23}\cdot\hat{e}_3 = \sigma_{2i}\cdot\hat{e}_i$$

$$\overset{3}{T} = \sigma_{31}\cdot\hat{e}_1 + \sigma_{32}\cdot\hat{e}_2 + \sigma_{33}\cdot\hat{e}_3 = \sigma_{3i}\cdot\hat{e}_i$$



$$\overset{\hat{n}}{T} = \overset{1}{T}\cdot n_1 + \overset{2}{T}\cdot n_2 + \overset{3}{T}\cdot n_3$$
$$= \overset{\hat{n}}{T}_1\cdot\hat{e}_1 + \overset{\hat{n}}{T}_2\cdot\hat{e}_2 + \overset{\hat{n}}{T}_3\cdot\hat{e}_3$$

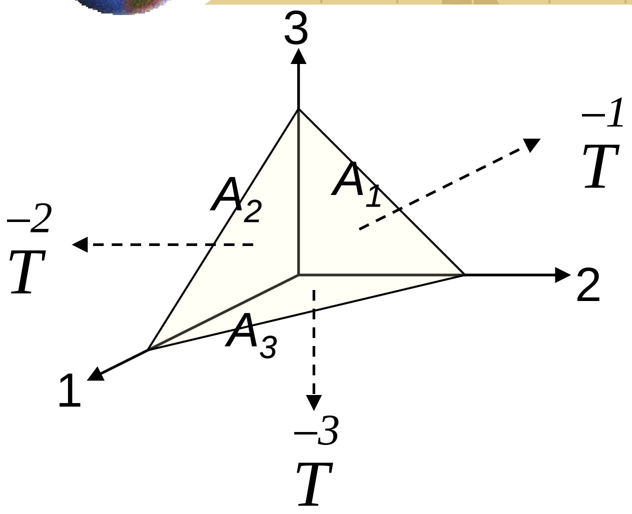
$$\overset{\hat{n}}{T}_1 = \sigma_{11}\cdot n_1 + \sigma_{21}\cdot n_2 + \sigma_{31}\cdot n_3$$

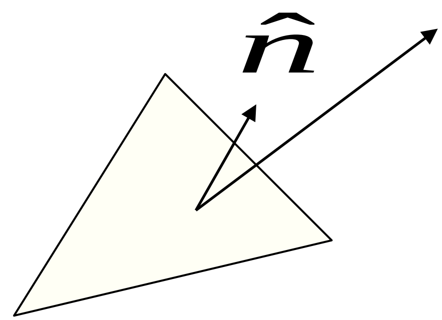
$$\overset{\hat{n}}{T}_2 = \sigma_{12}\cdot n_1 + \sigma_{22}\cdot n_2 + \sigma_{32}\cdot n_3$$

$$\overset{\hat{n}}{T}_3 = \sigma_{13}\cdot n_1 + \sigma_{23}\cdot n_2 + \sigma_{33}\cdot n_3$$

$$\overset{\hat{n}}{T}_i = \sigma_{ji}\cdot n_j = \sigma_{ij}\cdot n_j$$

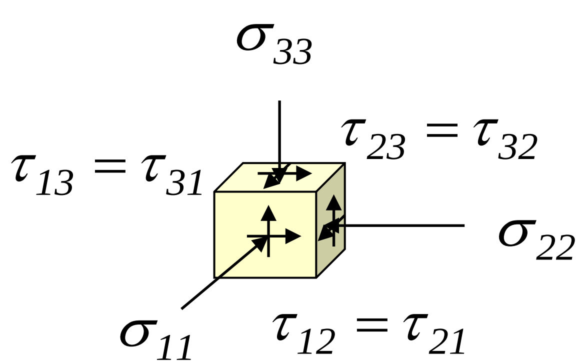
Three dimensional stress analysis





$$\overset{\hat{n}}{T} = \overset{1}{T} \cdot n_1 + \overset{2}{T} \cdot n_2 + \overset{3}{T} \cdot n_3$$

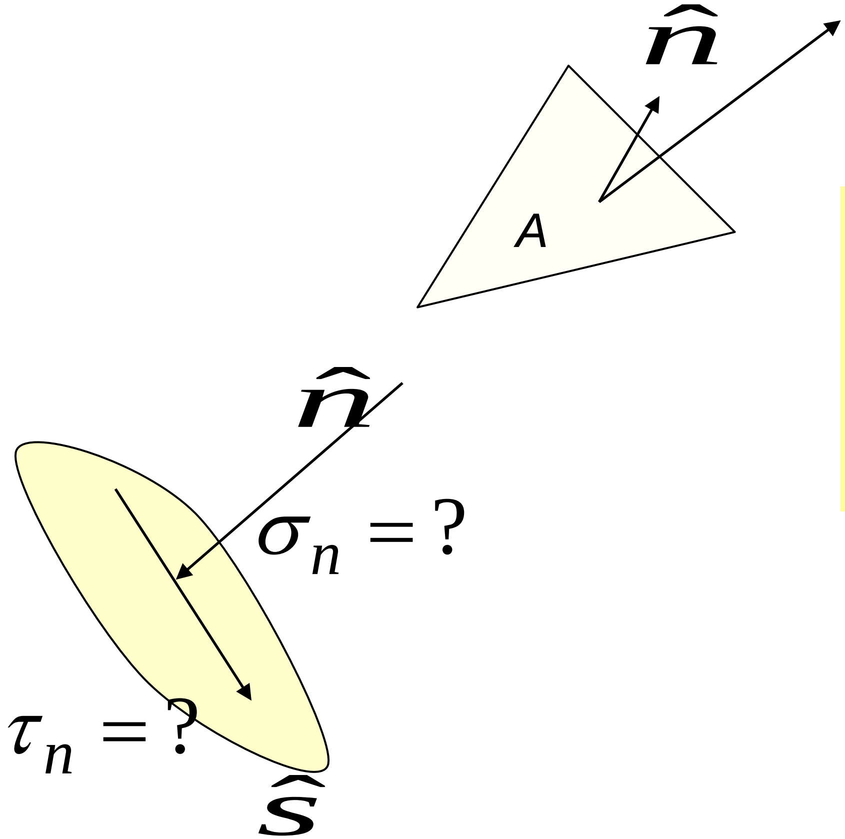
$$\overset{\hat{n}}{T}_i = \sigma_{ji} \cdot n_j = \sigma_{ij} \cdot n_j$$ **Eq.(2.8)**



In matrix form

$$\begin{bmatrix} \overset{\hat{n}}{T}_1 \\ \overset{\hat{n}}{T}_2 \\ \overset{\hat{n}}{T}_3 \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \cdot \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$ **Eq.(2.7)**

任意一平面上之 ***surface traction***
為該點之應力張量及該平面法線向量
之線性組合



$$\overset{\hat{n}}{T}_i = \sigma_{ji} \cdot n_j = \sigma_{ij} \cdot n_j$$

$$\sigma_n = \overset{\hat{n}}{T}_i \cdot n_i = \sigma_{ij} \cdot n_i \cdot n_j$$

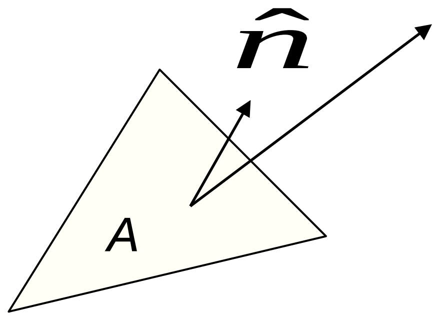
$$\tau_n = \overset{\hat{n}}{T}_i \cdot s_i = \sigma_{ij} \cdot n_i \cdot s_j$$

$$\left|\overset{\hat{n}}{T}_i\right|^2 = \sigma_n{}^2 + \tau_n{}^2$$

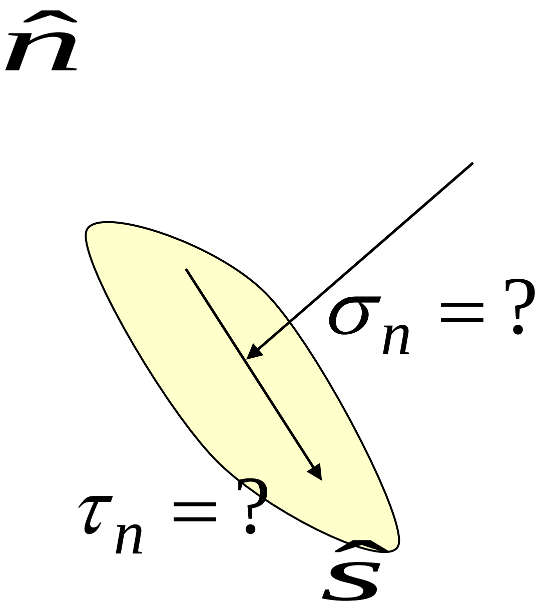
$$\tau_n{}^2 = \left|\overset{\hat{n}}{T}_i\right|^2 - \sigma_n{}^2$$

$$\left|\overset{\hat{n}}{T}_i\right|^2 = \overset{\hat{n}}{T}_i \cdot \overset{\hat{n}}{T}_i$$

Three dimensional stress analysis



$$\sigma_{ij} = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix}; \hat{n} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

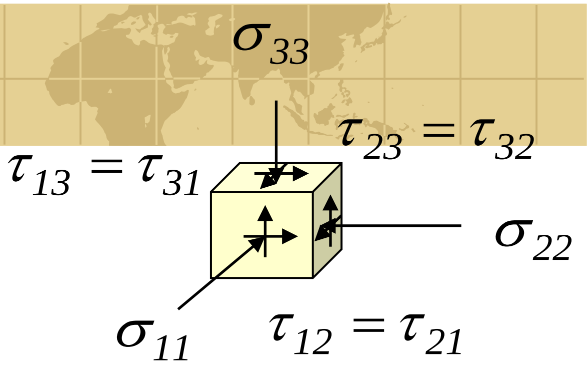


$$\overset{\hat{n}}{T}_i = \sigma_{ij} \cdot n_j = \left(\frac{6}{\sqrt{3}}, \frac{3}{\sqrt{3}}, \frac{4}{\sqrt{3}}\right)$$

$$\sigma_n = \overset{\hat{n}}{T}_i \cdot \hat{n}_i = \frac{13}{3}$$

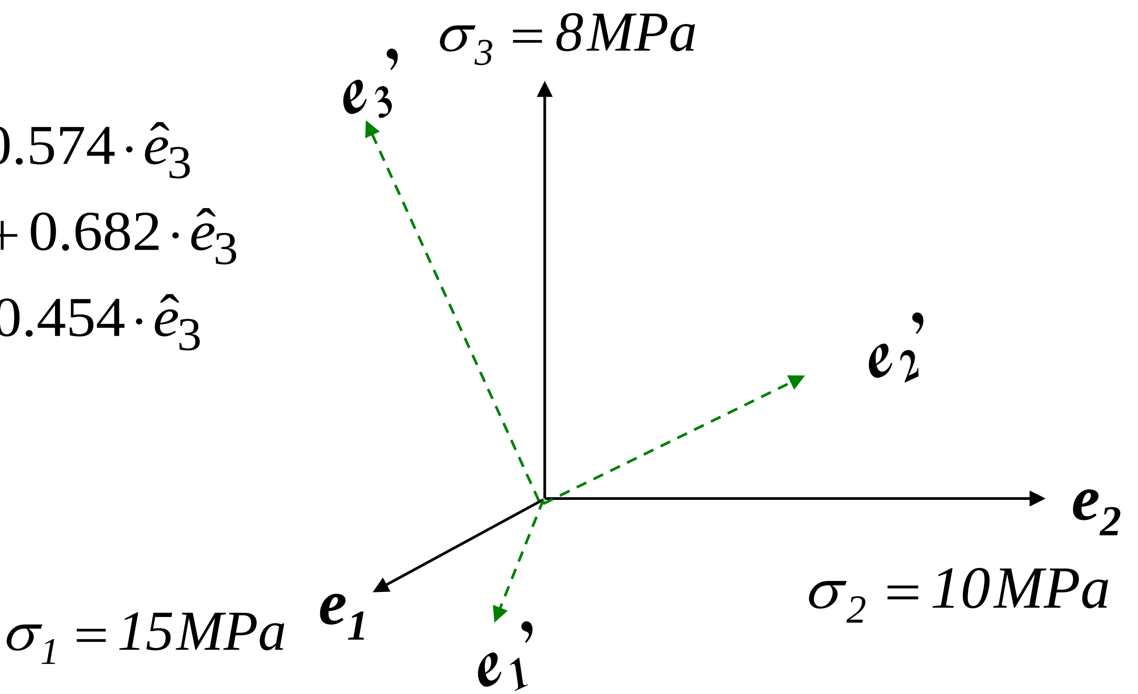
$$\tau_n^{\,2} = \left|\overset{\hat{n}}{T}_i\right|^2 - \sigma_n^{\,2} = \overset{\hat{n}}{T}_i \cdot \overset{\hat{n}}{T}_i - \left(\frac{13}{3}\right)^2 = \frac{61}{3} - \frac{169}{9} = \frac{14}{9}$$

Stress transformation



How to find the stress tensor in different coordinate system

$$\hat{e}_1' = 0.071 \cdot \hat{e}_1 + 0.816 \cdot \hat{e}_2 + 0.574 \cdot \hat{e}_3$$
$$\hat{e}_2' = -0.584 \cdot \hat{e}_1 - 0.440 \cdot \hat{e}_2 + 0.682 \cdot \hat{e}_3$$
$$\hat{e}_3' = 0.808 \cdot \hat{e}_1 - 0.377 \cdot \hat{e}_2 + 0.454 \cdot \hat{e}_3$$



座標轉換

$$x_1' = \overset{1}{x_1 \cdot \cos\theta} + \overset{2,3}{x_2 \sin\theta}$$

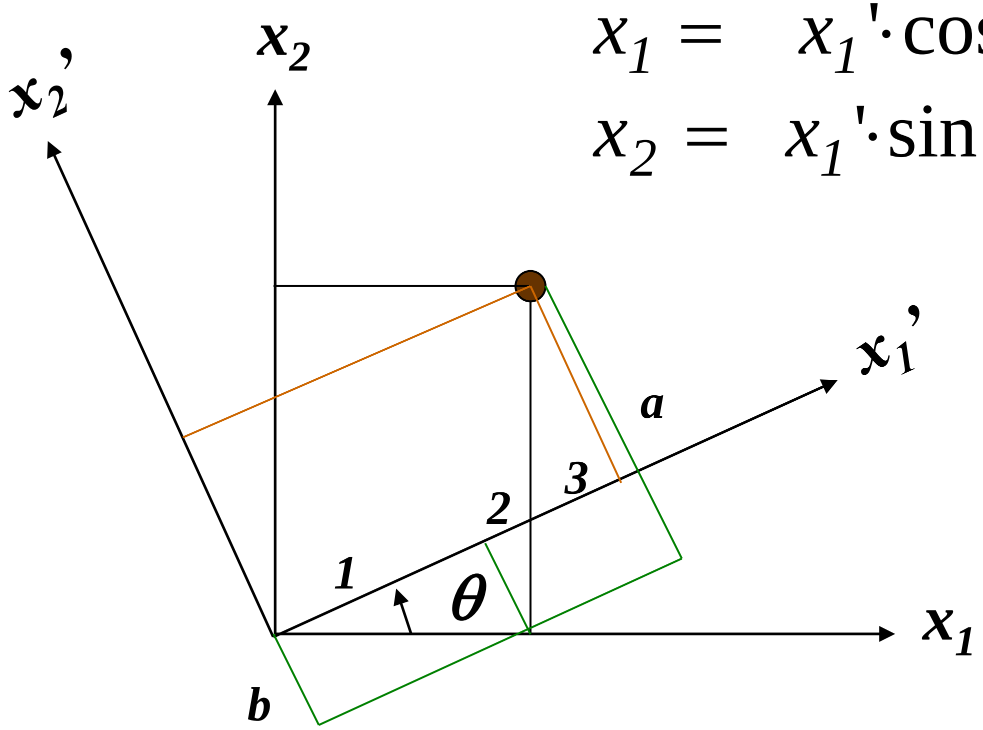
$$x_2' = -\underset{b}{x_1 \cdot \sin\theta} + \underset{a}{x_2 \cos\theta}$$

$$x_i' = \beta_{ij} \cdot x_j \quad (i=1,2)$$

$$x_1 = x_1' \cdot \cos\theta - x_2' \sin\theta$$

$$x_2 = x_1' \cdot \sin\theta + x_2' \cos\theta$$

$$x_i = \beta_{ji} \cdot x_j'$$



$$\beta_{ij} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$\beta_{ji} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

座標轉換

$$\beta_{ij} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \quad \beta_{ji} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \Longrightarrow \beta_{ji} = \beta_{ij}{}^{T}$$

$$x_i' = \beta_{ij} \cdot x_j \qquad x_i = \beta_{ji} \cdot x_j' \Longrightarrow \beta_{ji} = \beta_{ij}{}^{-1}$$

$$\beta_{ij}{}^{-1} = \beta_{ij}{}^{T} \Longrightarrow$$ 正交矩陣

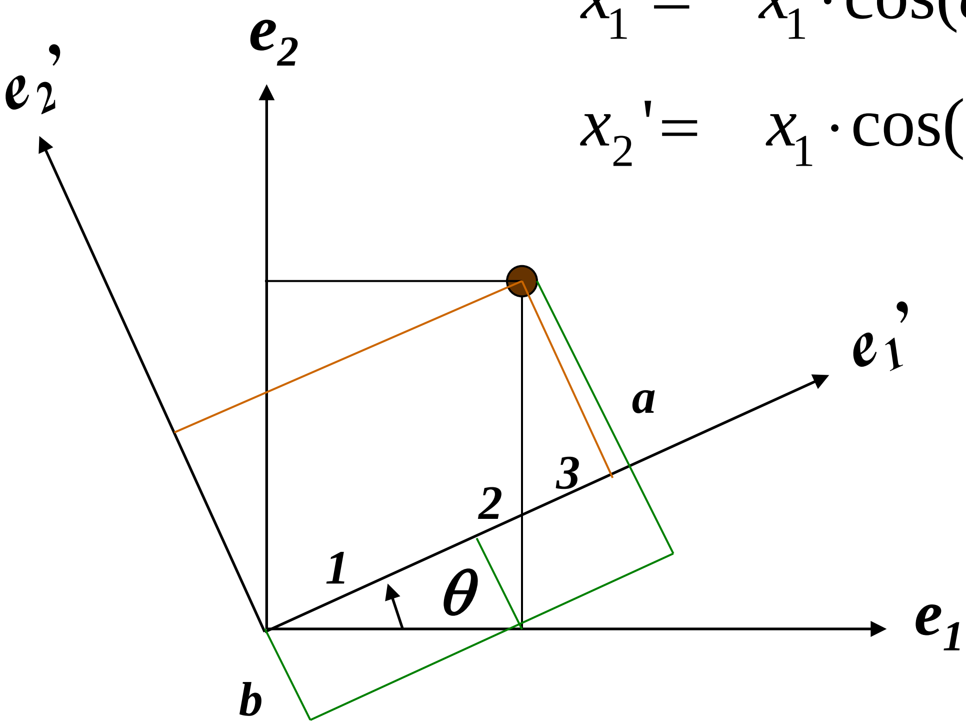
$\Longrightarrow$ 座標轉換為一種正交轉換

座標轉換

$$x_1' = \quad x_1 \cdot \cos\theta + x_2 \sin\theta$$
$$x_2' = -x_1 \cdot \sin\theta + x_2 \cos\theta$$

$$x_i' = \beta_{ij} \cdot x_j$$

$$x_1' = \quad x_1 \cdot \cos(e_1', e_1) + x_2 \cos(e_1', e_2)$$
$$x_2' = \quad x_1 \cdot \cos(e_2', e_1) + x_2 \cos(e_2', e_2)$$



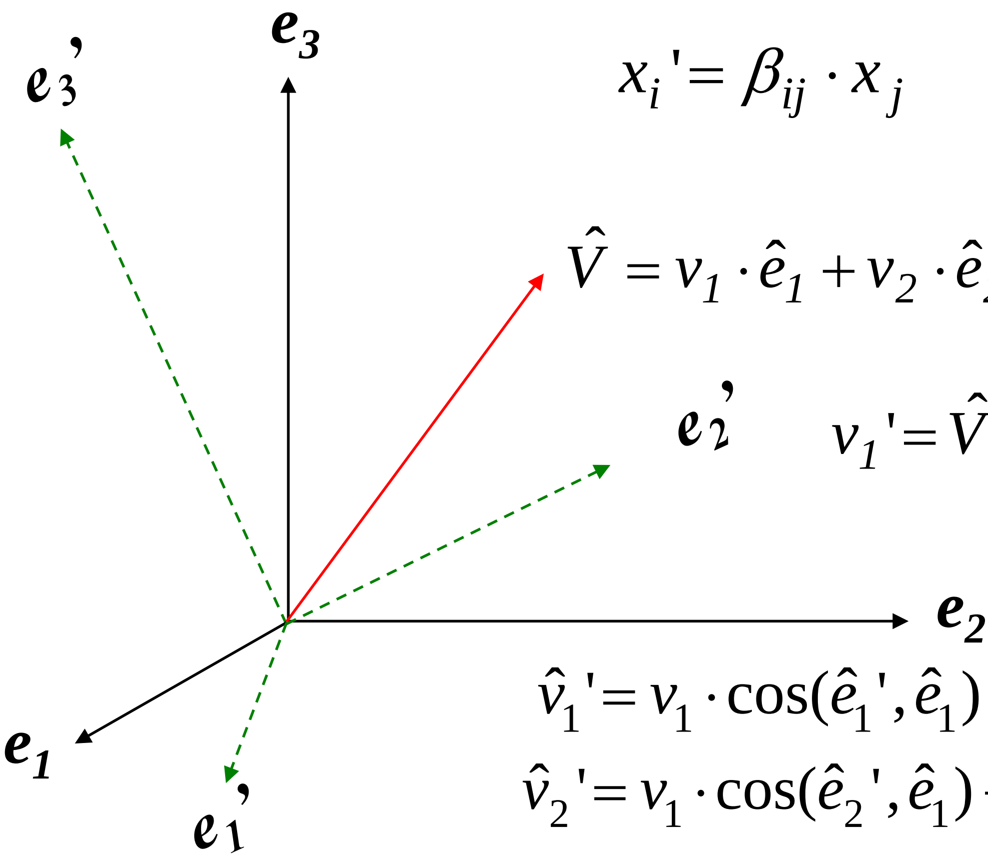
$$\beta_{ij} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$
$$= \begin{bmatrix} \cos(\hat{e}_1', \hat{e}_1) & \cos(\hat{e}_1', \hat{e}_2) \\ \cos(\hat{e}_2', \hat{e}_1) & \cos(\hat{e}_2', \hat{e}_2) \end{bmatrix}$$

座標轉換

$$x_1' = x_1 \cdot \cos(e_1', e_1) + x_2 \cos(e_1', e_2)$$

$$x_2' = x_1 \cdot \cos(e_2', e_1) + x_2 \cos(e_2', e_2)$$

$$x_i' = \beta_{ij} \cdot x_j \qquad \beta_{ij} = \begin{bmatrix} \cos(\hat{e}_1', \hat{e}_1) & \cos(\hat{e}_1', \hat{e}_2) \\ \cos(\hat{e}_2', \hat{e}_1) & \cos(\hat{e}_2', \hat{e}_2) \end{bmatrix}$$



$$\hat{V} = v_1 \cdot \hat{e}_1 + v_2 \cdot \hat{e}_2 + v_3 \cdot \hat{e}_3 = v_1' \cdot \hat{e}_1' + v_2' \cdot \hat{e}_2' + v_3' \cdot \hat{e}_3'$$

$$v_1' = \hat{V} \cdot \hat{e}_1'; \quad v_2' = \hat{V} \cdot \hat{e}_2'; \quad v_3' = \hat{V} \cdot \hat{e}_3'$$

$$\hat{v}_1' = v_1 \cdot \cos(\hat{e}_1', \hat{e}_1) + v_2 \cdot \cos(\hat{e}_1', \hat{e}_2) + v_3 \cdot \cos(\hat{e}_1', \hat{e}_3)$$

$$\hat{v}_2' = v_1 \cdot \cos(\hat{e}_2', \hat{e}_1) + v_2 \cdot \cos(\hat{e}_2', \hat{e}_2) + v_3 \cdot \cos(\hat{e}_2', \hat{e}_3)$$

$$\hat{v}_3' = v_1 \cdot \cos(\hat{e}_3', \hat{e}_1) + v_2 \cdot \cos(\hat{e}_3', \hat{e}_1) + v_3 \cdot \cos(\hat{e}_3', \hat{e}_3)$$