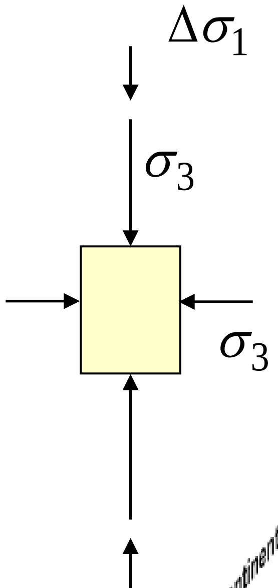
1 Stress and infinitesimal strain

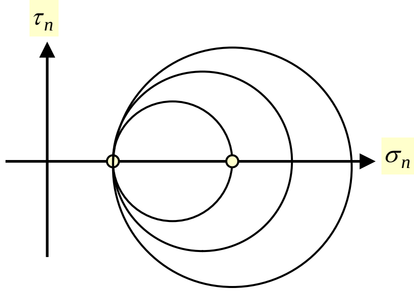
Topic 1 Stress

Topic 2 Strain

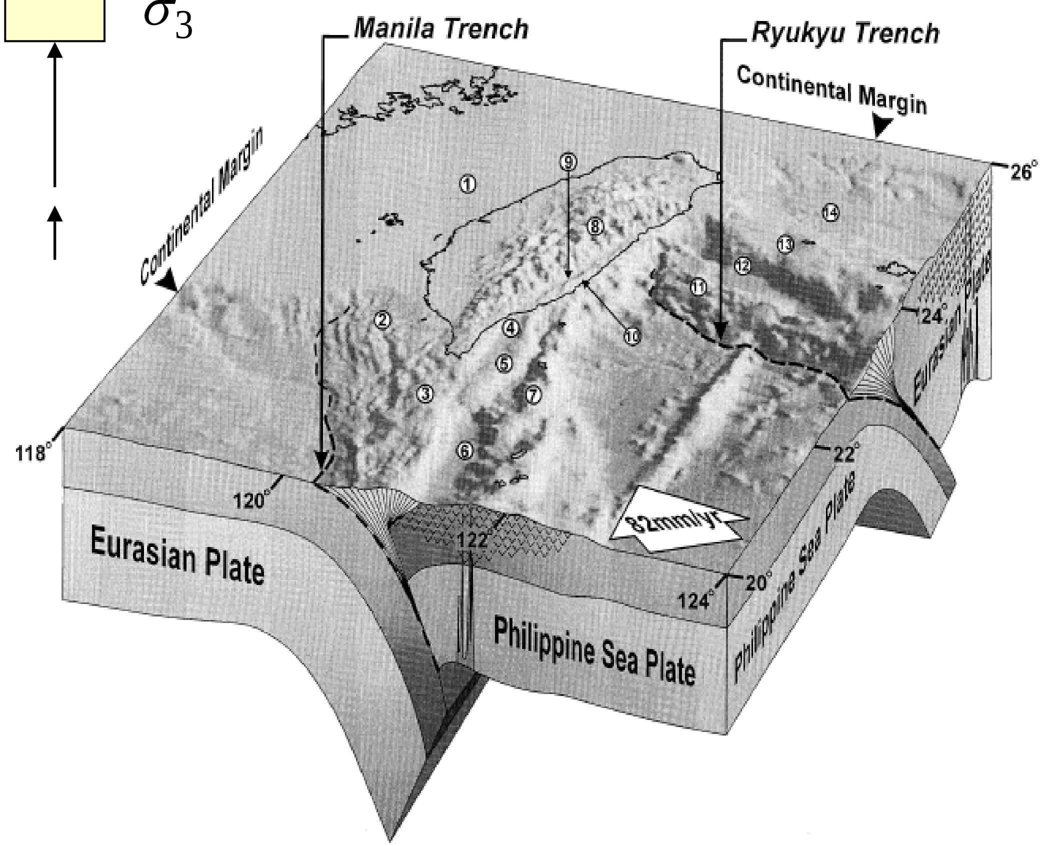
Topic 3 Elastic constants



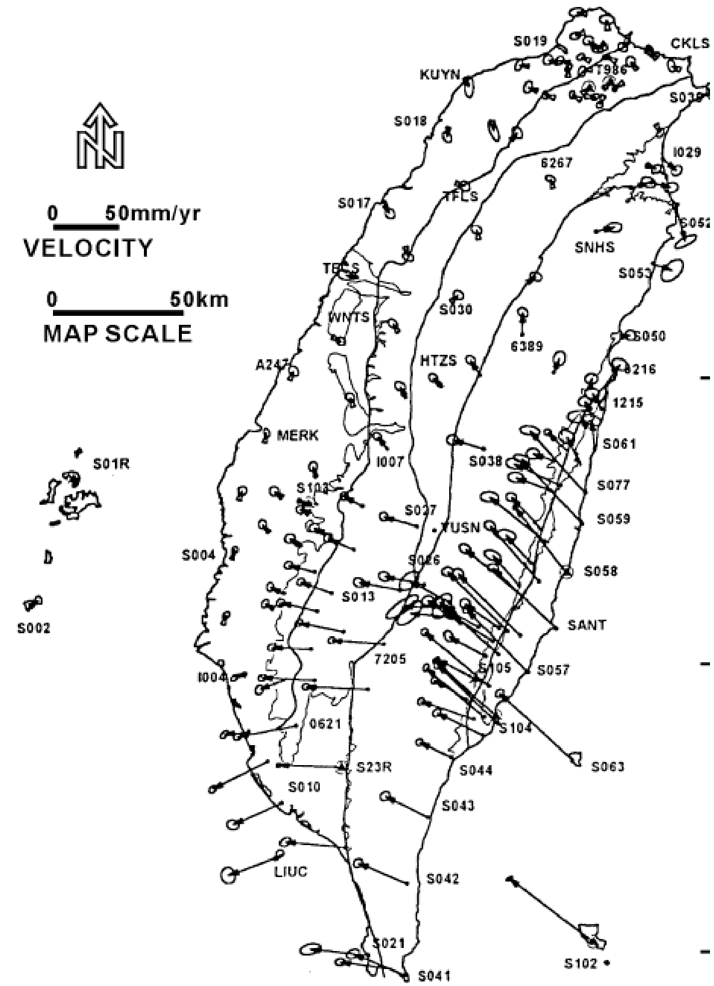
$\Delta\sigma_1$
σ_3
σ_3



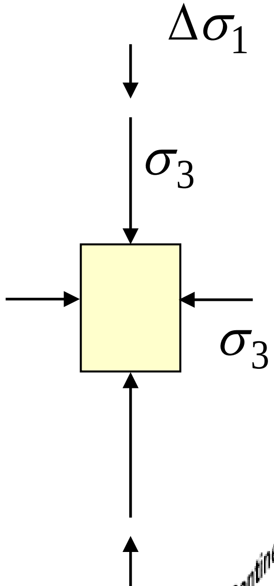
τ_n
σ_n



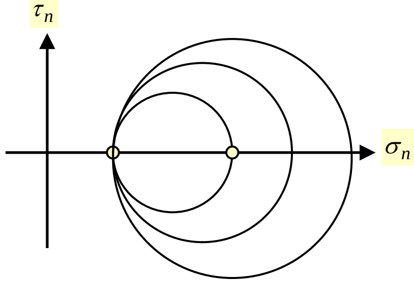
Manila Trench
Ryukyu Trench
Continental Margin
Continental Margin
Eurasian Plate
Philippine Sea Plate
Philippine Sea Plate
Eurasian Plate
82mm/yr
118°
120°
122°
124°
20°
22°
24°
26°



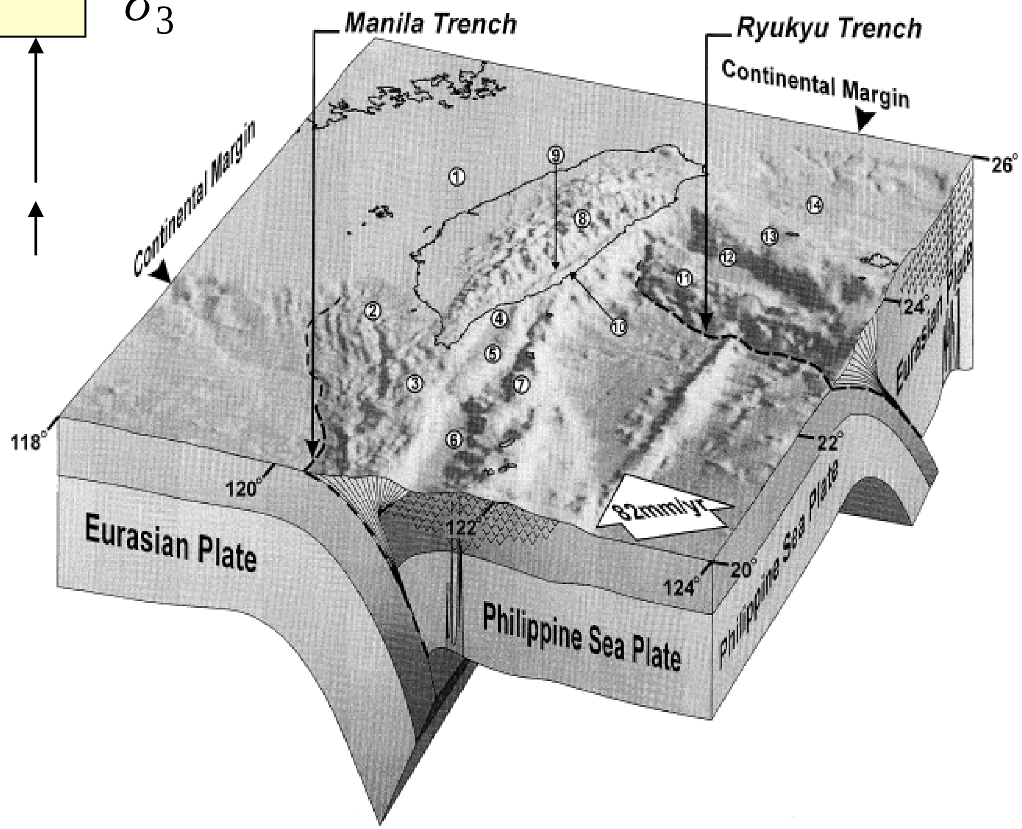
0 50mm/yr
VELOCITY
0 50km
MAP SCALE
S019
CKLS
KUYN
S018
I029
6267
S017
TFLS
SNHS
S053
S030
6389
S050
A247
HTZS
S216
1215
MERK
I007
S038
S061
S01R
S077
S027
TUSN
S059
S004
S026
S058
S013
SANT
S002
7205
S105
S057
I004
0621
S104
S063
S23R
S044
S010
S043
LIUC
S042
S021
S041
S102



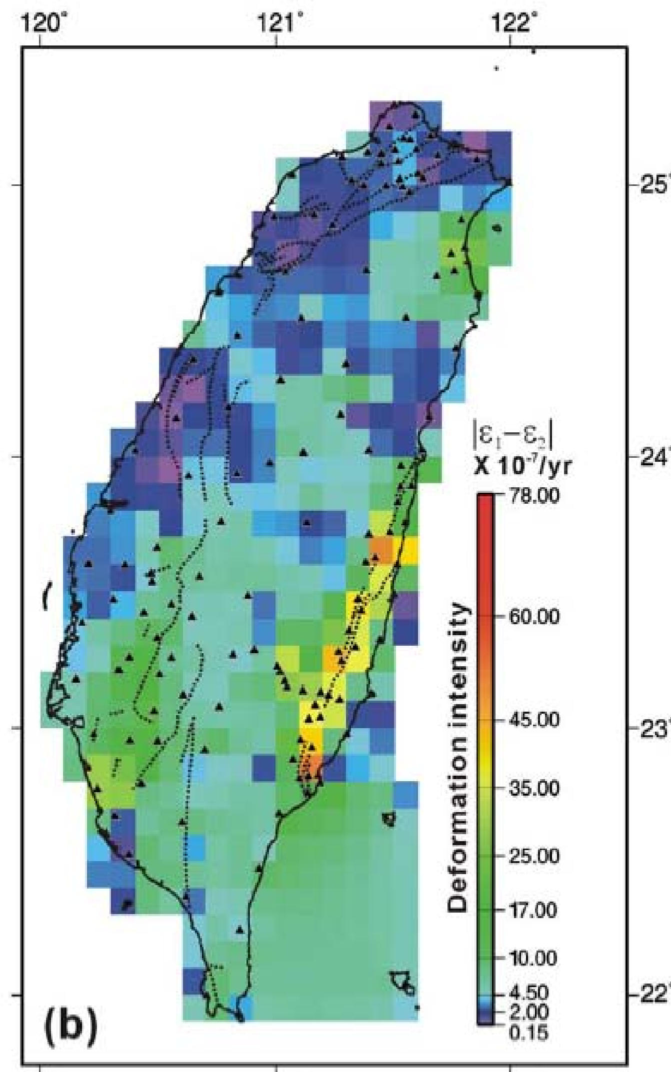
$\Delta\sigma_1$
σ_3
σ_3



τ_n
σ_n

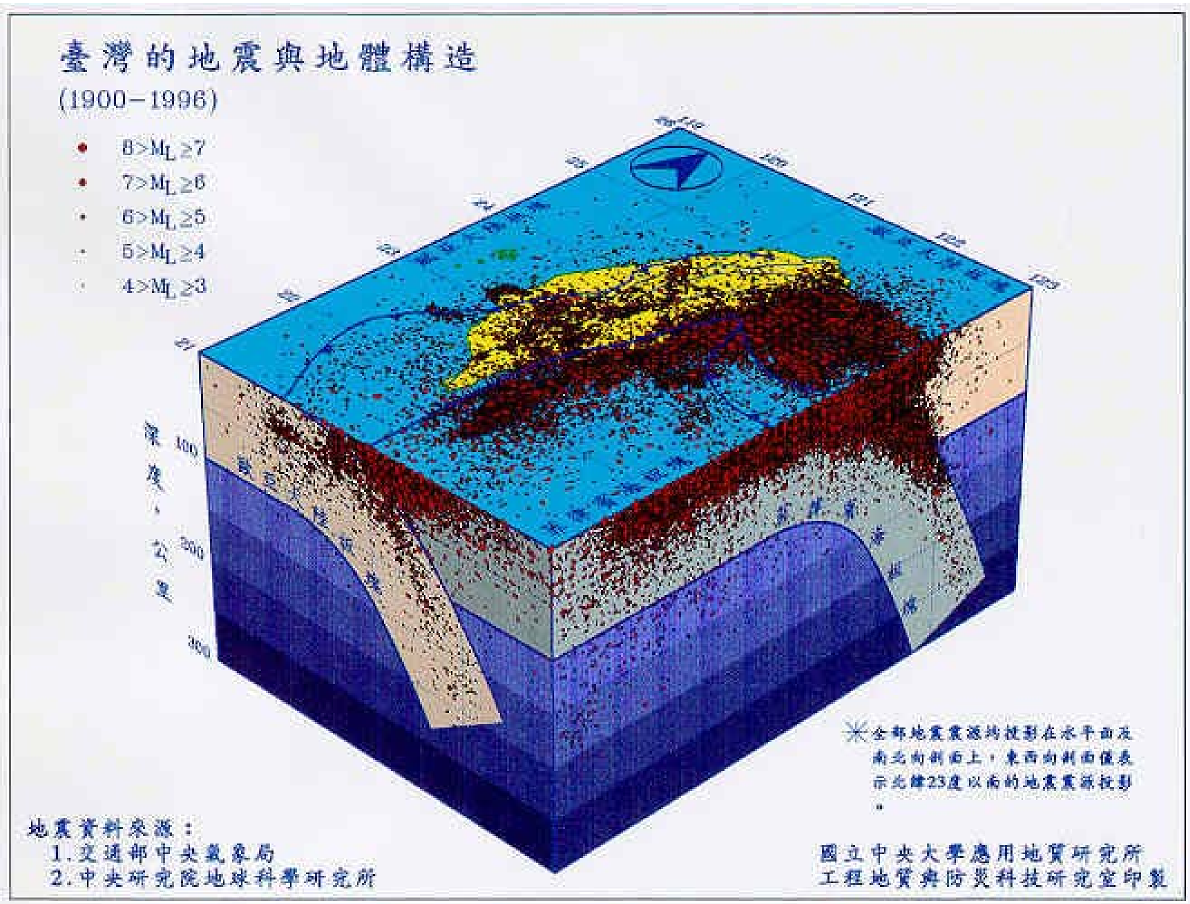


Manila Trench
Ryukyu Trench
Continental Margin
Continental Margin
26°
24°
22°
20°
118°
120°
122
124°
82mm/yr
Eurasian Plate
Philippine Sea Plate
Philippine Sea Plate
Eurasian Plate

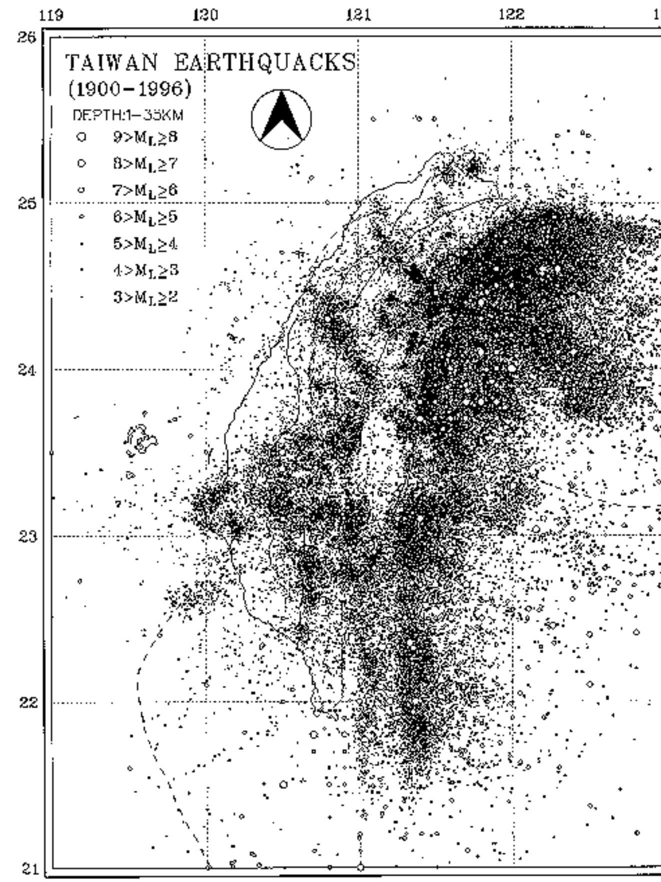


120°
121°
122°
25°
24°
23°
22°
$|\varepsilon_1-\varepsilon_2|$
X 10^{-7}/yr
Deformation intensity
78.00
60.00
45.00
35.00
25.00
17.00
10.00
4.50
2.00
0.15
(b)

Earthquake



圖十四 台灣地區的地體構造及地震分佈圖

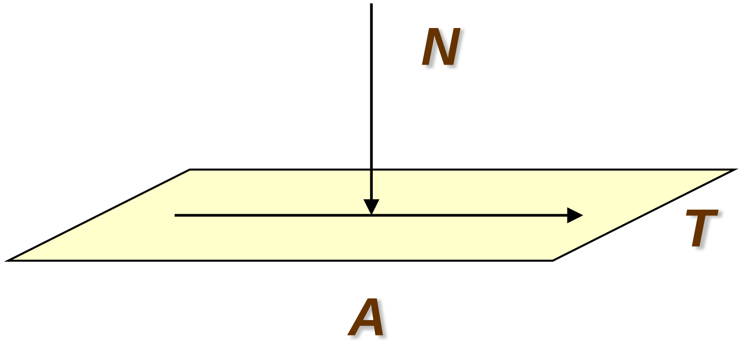


圖十三 台灣地區淺源地震震央分佈圖

What is stress

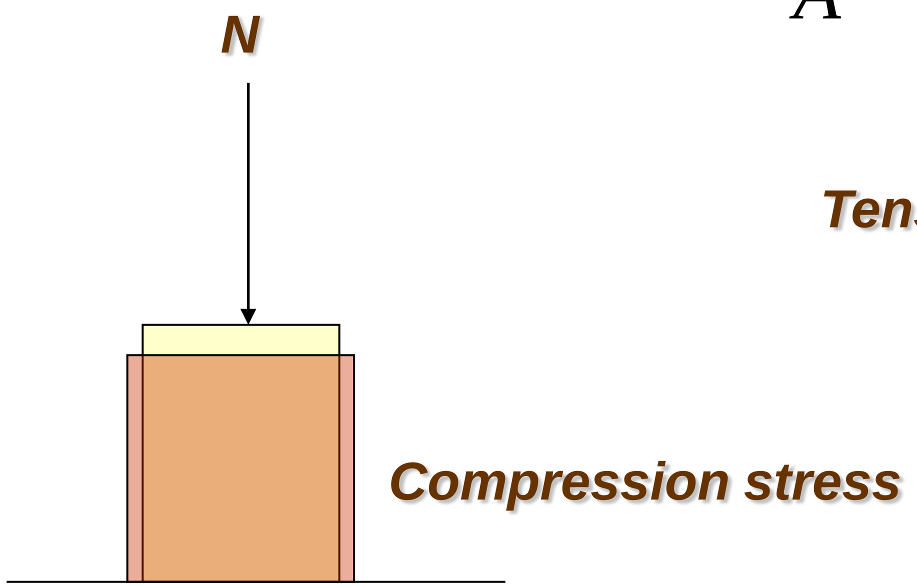
- Normal stress and shear stress
  - Normal force and shear force per unit area

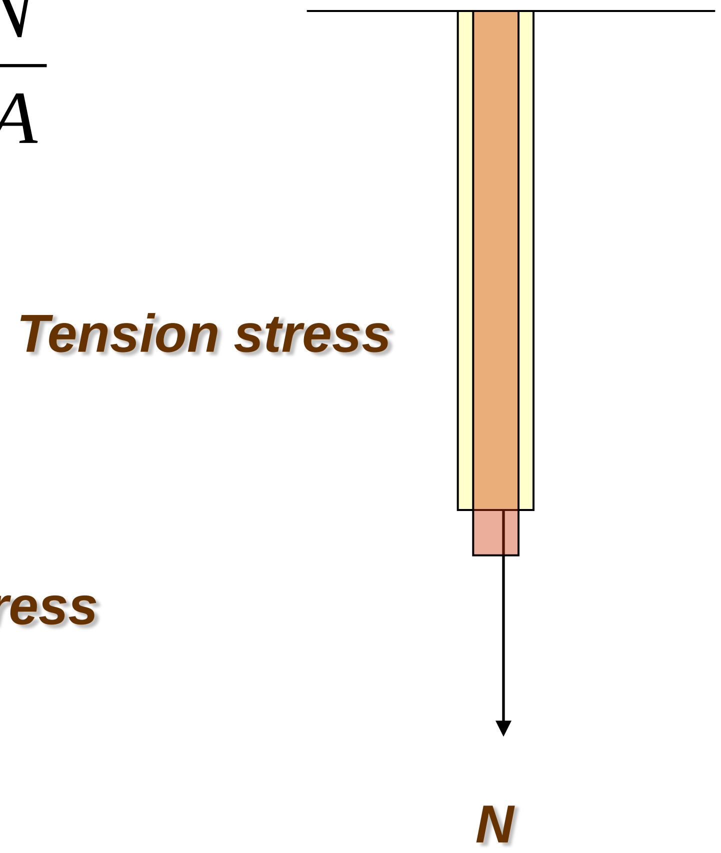
$$\sigma_n = \frac{N}{A} \qquad \tau_n = \frac{T}{A}$$



What is stress

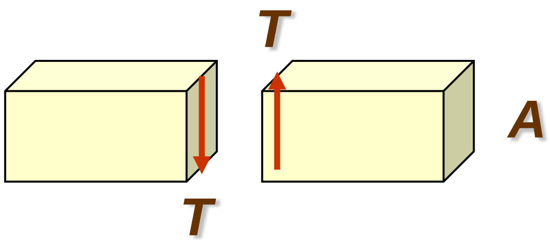
- Normal stress $\sigma_n = \dfrac{N}{A}$





What is stress

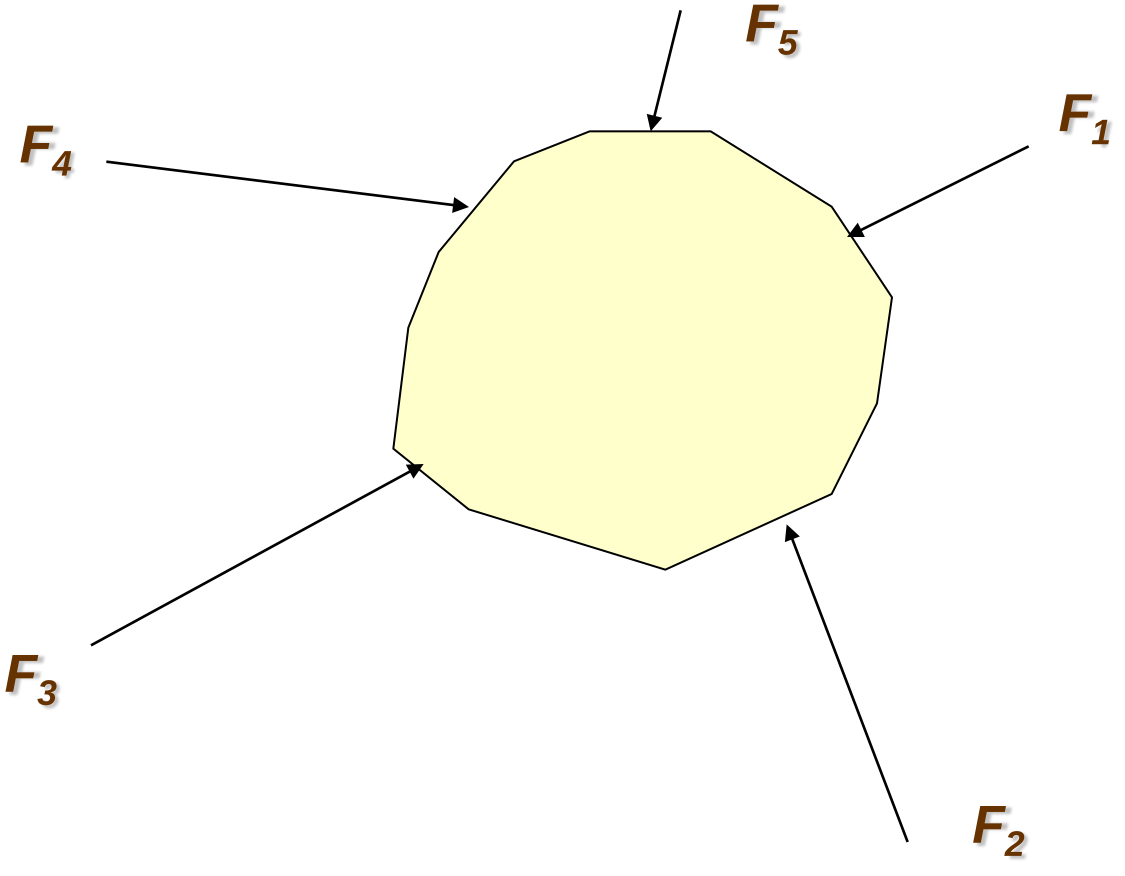
- Shear stress $$\tau_n = \frac{T}{A}$$



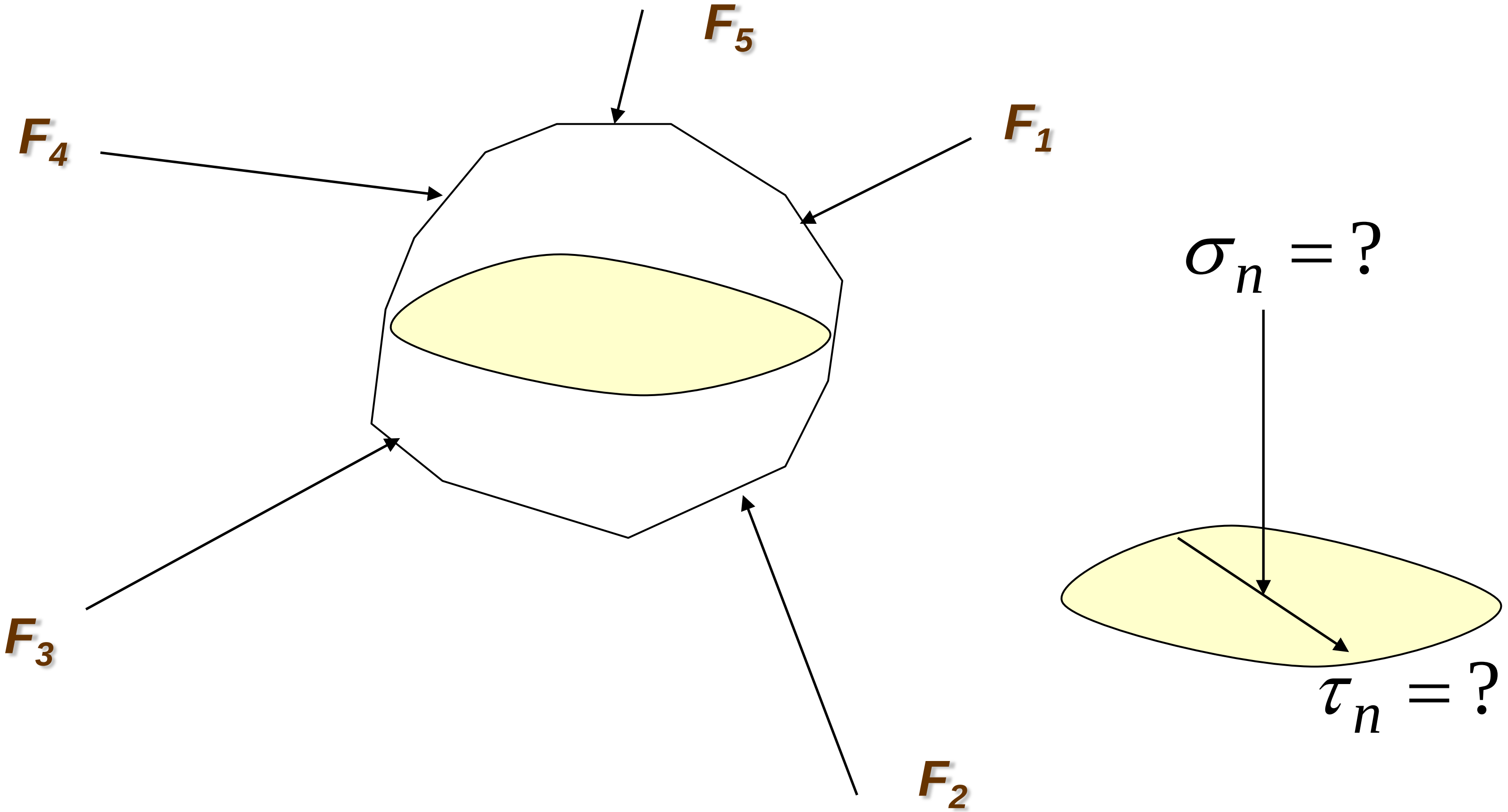
What is stress

- Force is a vector
- Stress ?

Stress ?

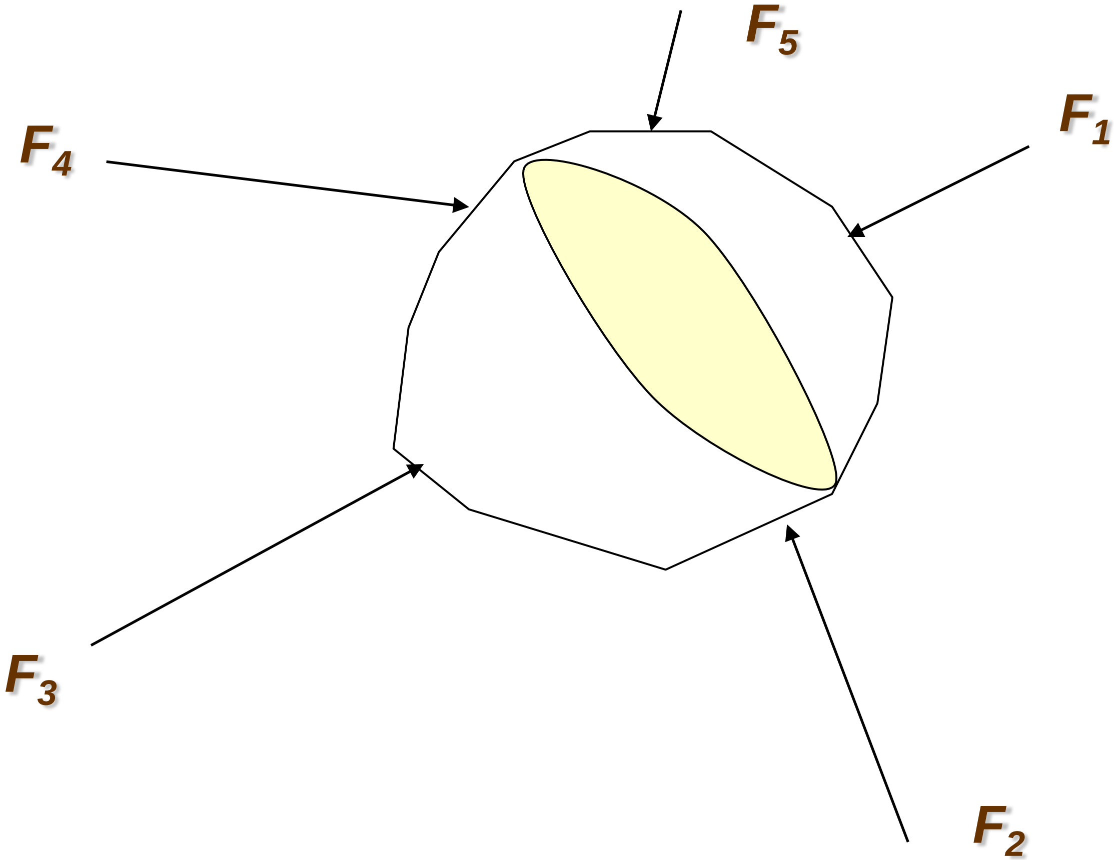


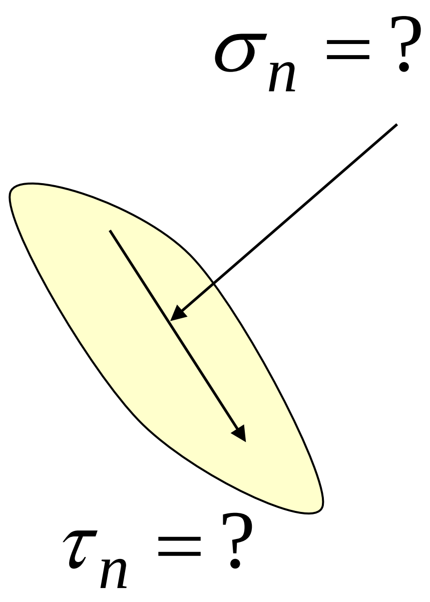
Stress ?



Stress ?

Surface traction :
Varied with the normal vector of concerned surface, tx,ty,tz

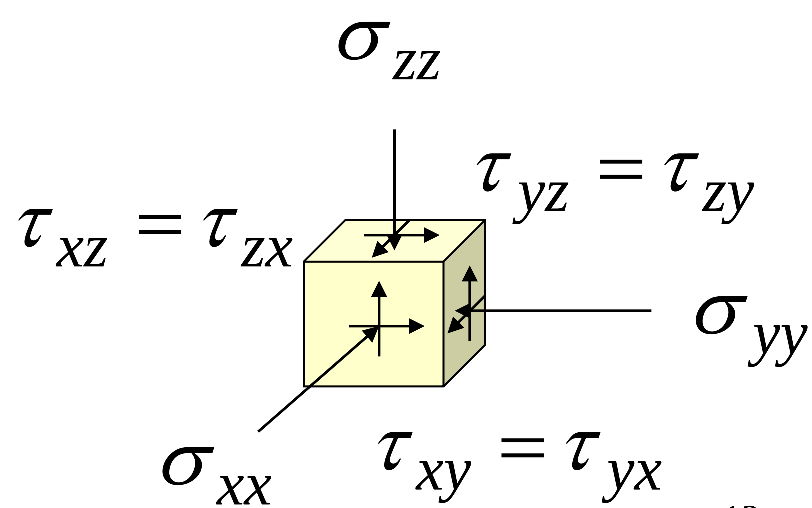




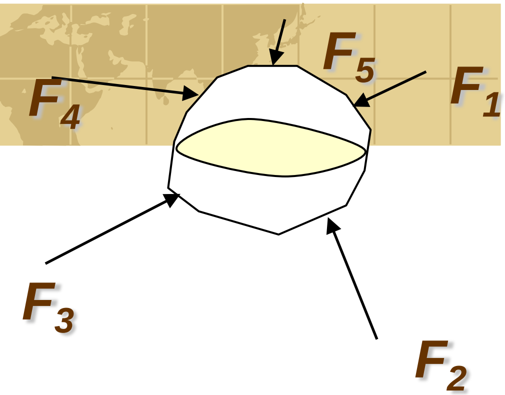
What is stress

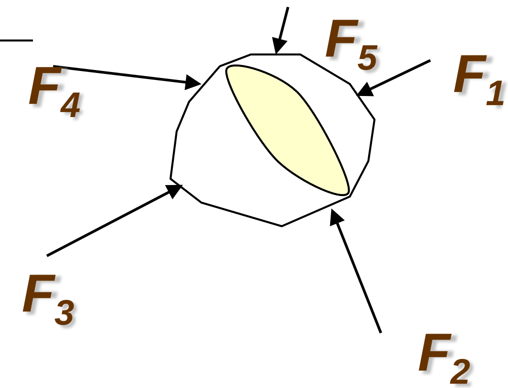
- Force is a vector
- Stress will varied with concerned plane
- Stress is a tensor
  - Defined by three surface tractions (vector) at three orthogonal plane

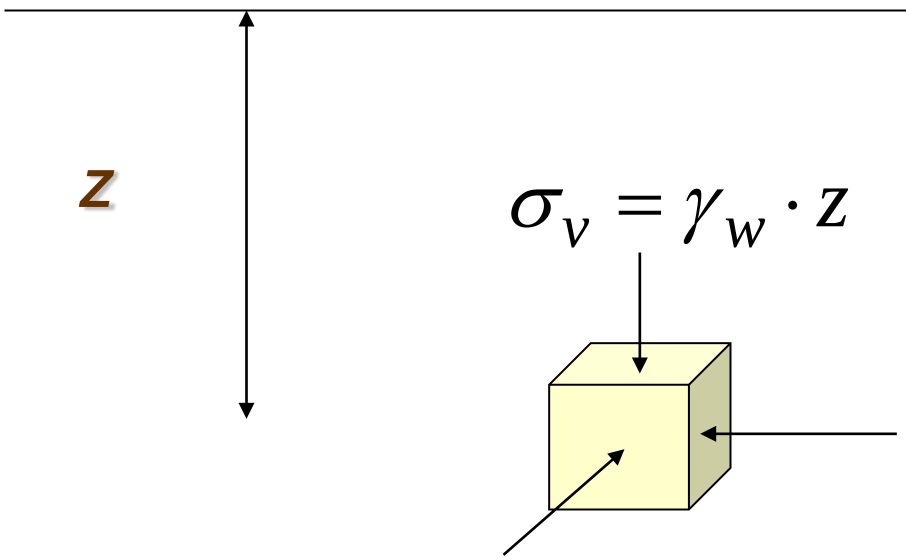
$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$



Stress ?







$$\sigma_v = \gamma_w \cdot z$$

$$\sigma_h = \gamma_w \cdot z$$

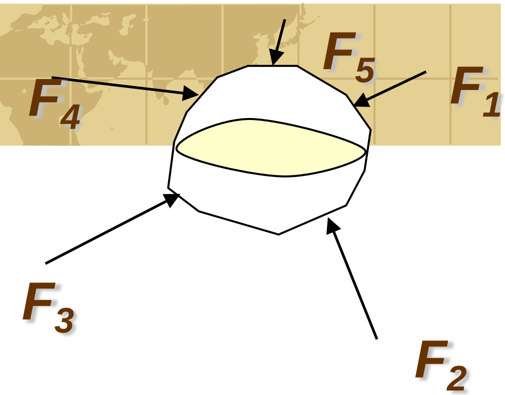
$$\sigma_h = \gamma_w \cdot z$$

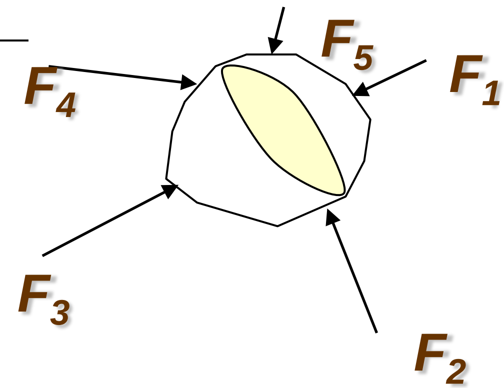
Shear stress at vertical and horizontal plane = ?

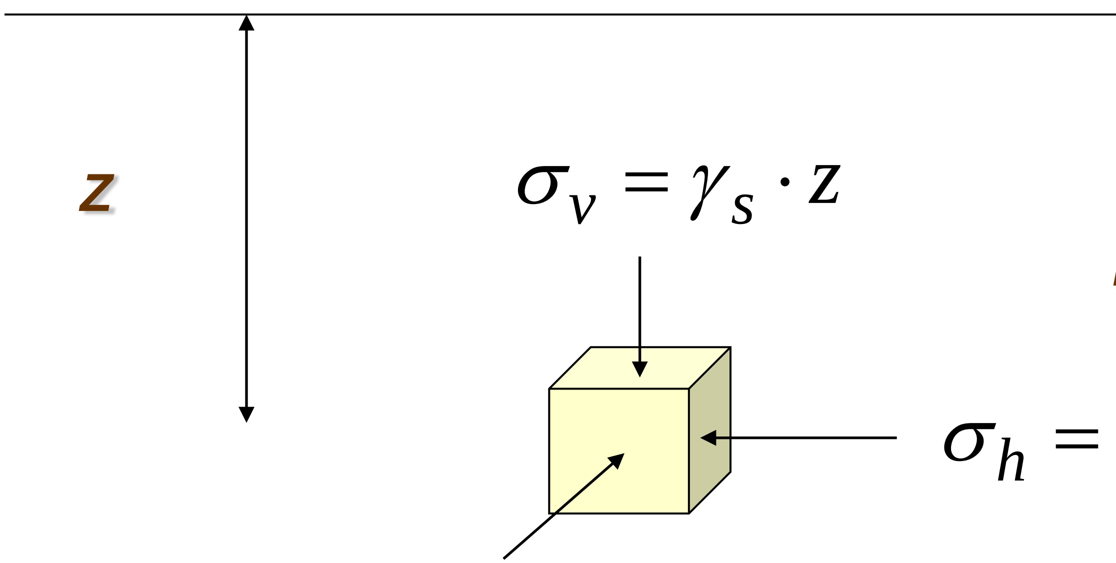
Normal stress at any incline plane = ?

Hydrostatic pressure!

Stress ?







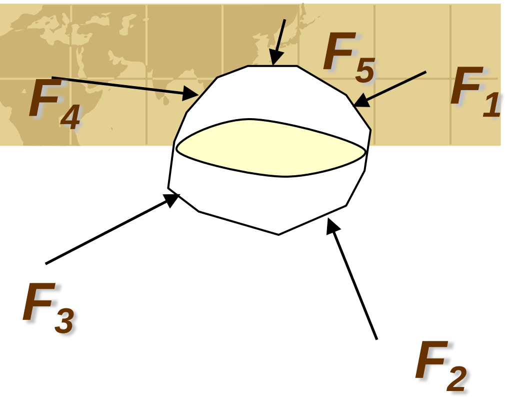
$$\sigma_v = \gamma_s \cdot z$$

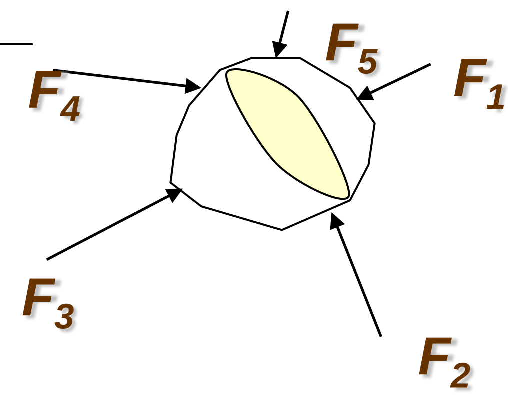
$$\sigma_h = k_o \cdot \gamma_s \cdot z$$

$$\sigma_h = k_o \cdot \gamma_s \cdot z$$

Shear stress at vertical and horizontal plane = ?

Stress ?



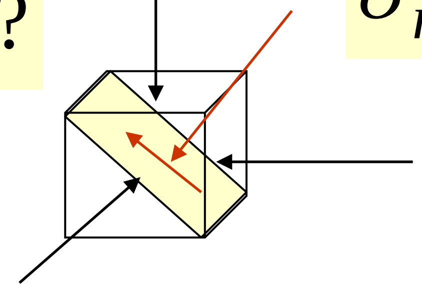


z

$\sigma_v = \gamma_s \cdot z$

$\tau_n = ?$

$\sigma_n = ?$

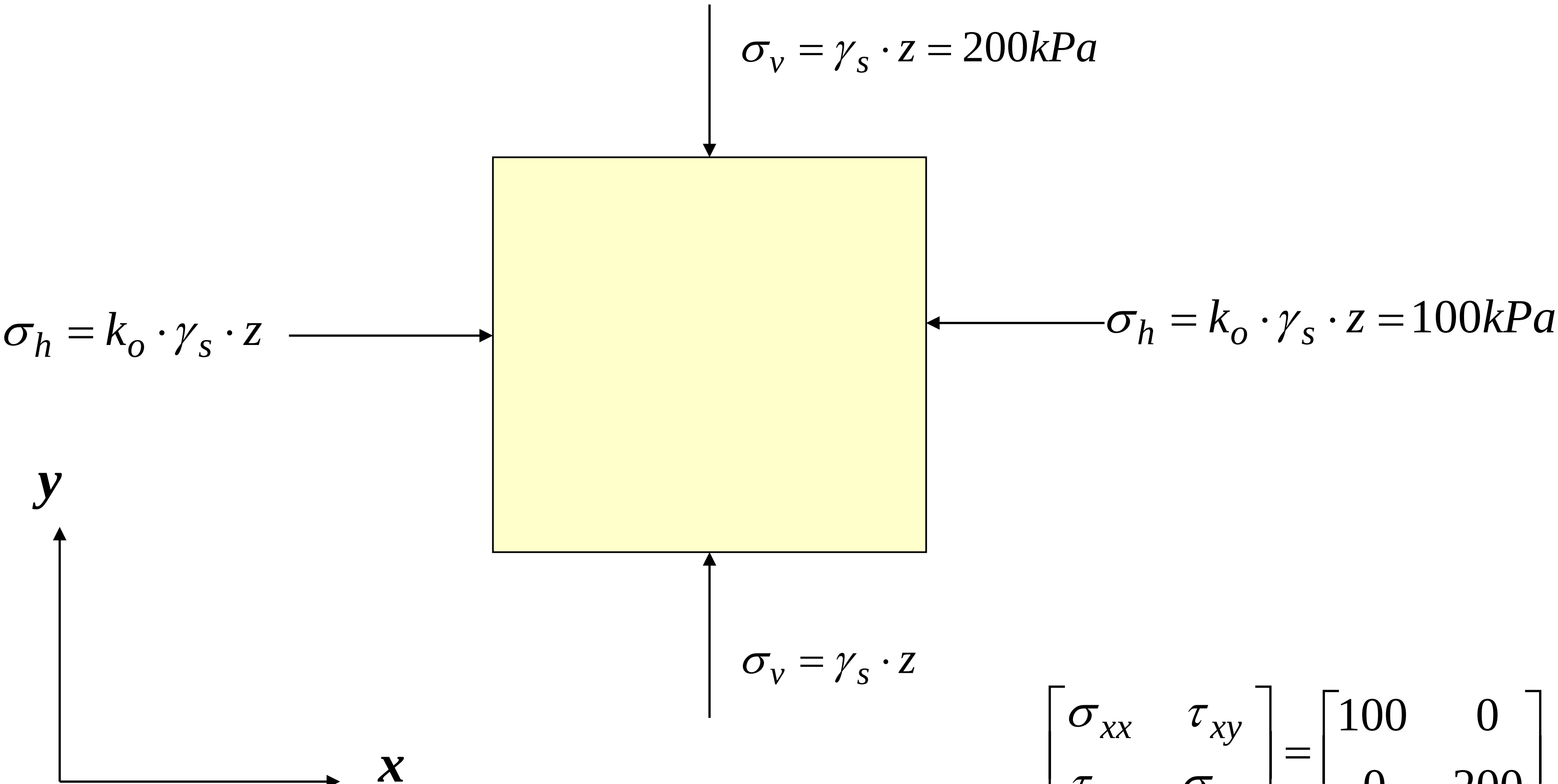


$\sigma_h = k_o \cdot \gamma_s \cdot z$

$\sigma_h = k_o \cdot \gamma_s \cdot z$

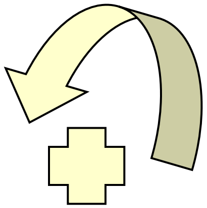
Normal stress and shear stress at a incline plane?

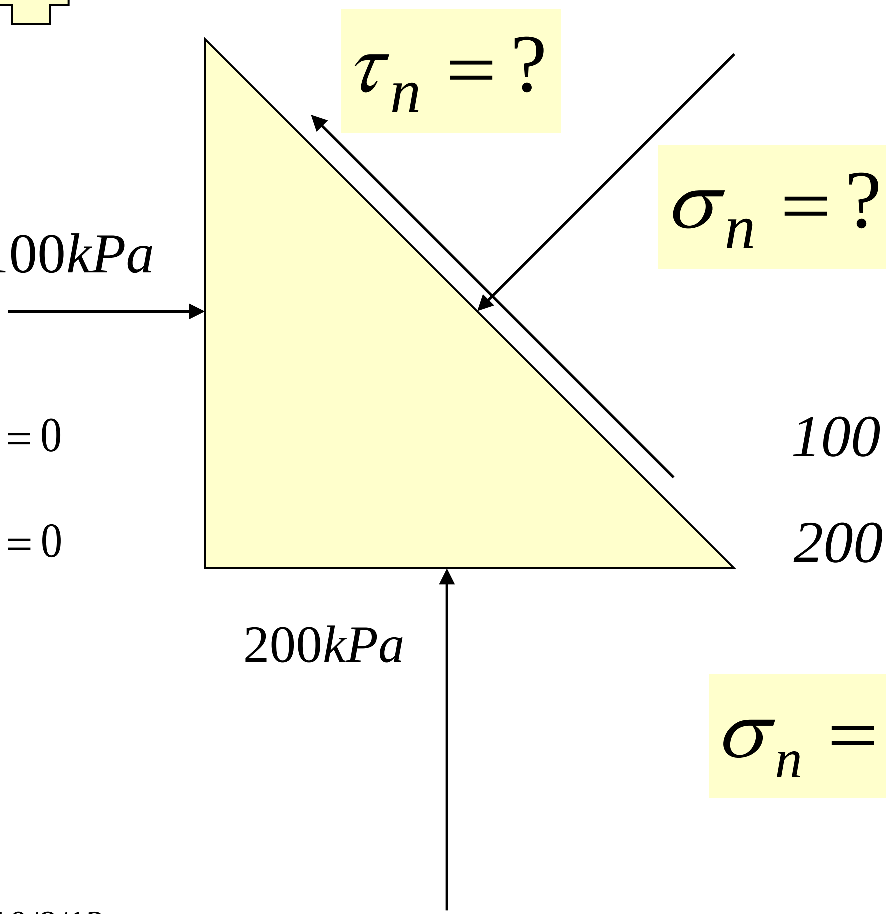
Two dimensional stress

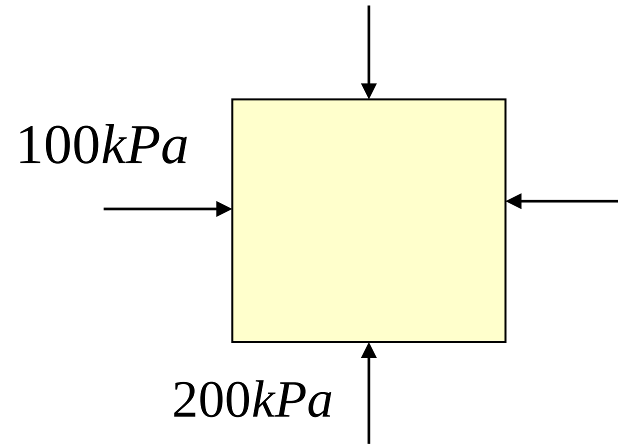


$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{bmatrix} = \begin{bmatrix} 100 & 0 \\ 0 & 200 \end{bmatrix}$$

Two dimensional stress







$$\sum F_x = 0$$

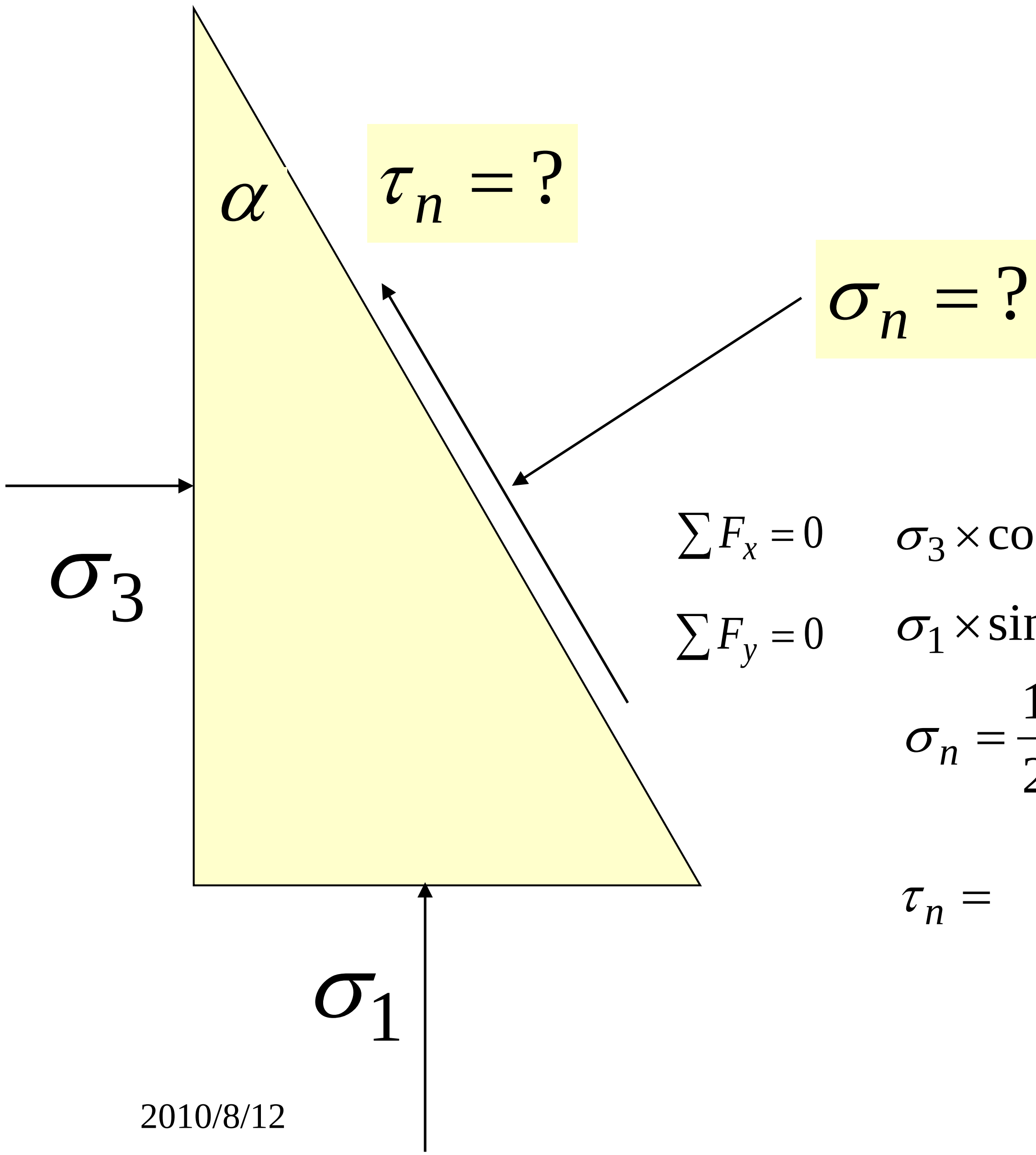
$$\sum F_y = 0$$

$$100 \times 1 = \sigma_n \times \sqrt{2} / \sqrt{2} + \tau_n \times \sqrt{2} / \sqrt{2}$$

$$200 \times 1 = \sigma_n \times \sqrt{2} / \sqrt{2} - \tau_n \times \sqrt{2} / \sqrt{2}$$

$$\sigma_n = 150kPa \quad \tau_n = -50kPa$$

Two dimensional stress



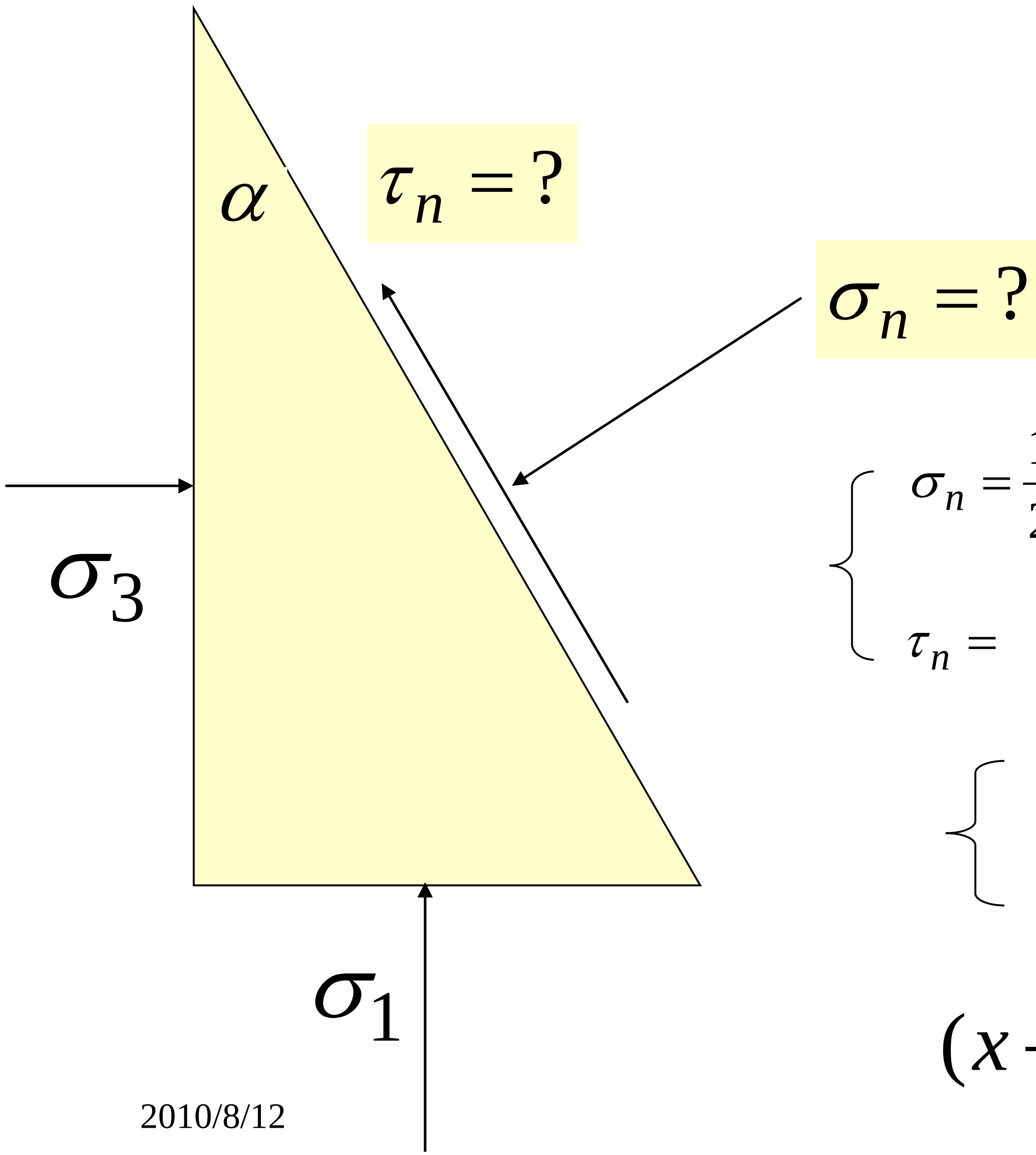
$$\sum F_x = 0 \qquad \sigma_3 \times \cos\alpha = \sigma_n \times 1 \times \cos\alpha + \tau_n \times 1 \times \sin\alpha$$

$$\sum F_y = 0 \qquad \sigma_1 \times \sin\alpha = \sigma_n \times 1 \times \sin\alpha - \tau_n \times 1 \times \cos\alpha$$

$$\sigma_n = \frac{1}{2}(\sigma_1 + \sigma_3) - \frac{1}{2}(\sigma_1 - \sigma_3) \times \cos 2\alpha$$

$$\tau_n = \qquad -\frac{1}{2}(\sigma_1 - \sigma_3) \times \sin 2\alpha$$

Two dimensional stress



$$\begin{cases} \sigma_n = \frac{1}{2}(\sigma_1 + \sigma_3) - \frac{1}{2}(\sigma_1 - \sigma_3) \times \cos 2\alpha \\ \tau_n = -\frac{1}{2}(\sigma_1 - \sigma_3) \times \sin 2\alpha \end{cases}$$

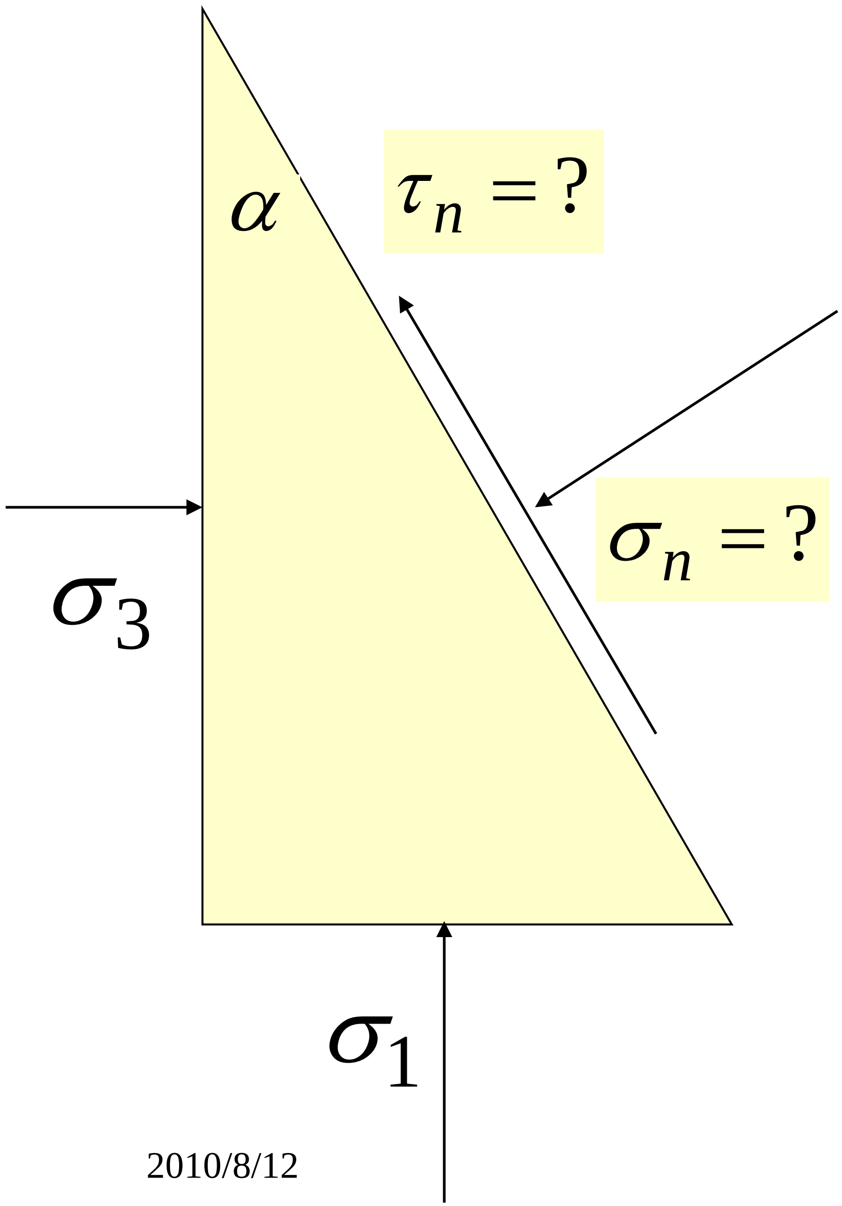
$$\begin{cases} x = x_o - R \times \cos 2\alpha \\ y = -R \times \sin 2\alpha \end{cases}$$

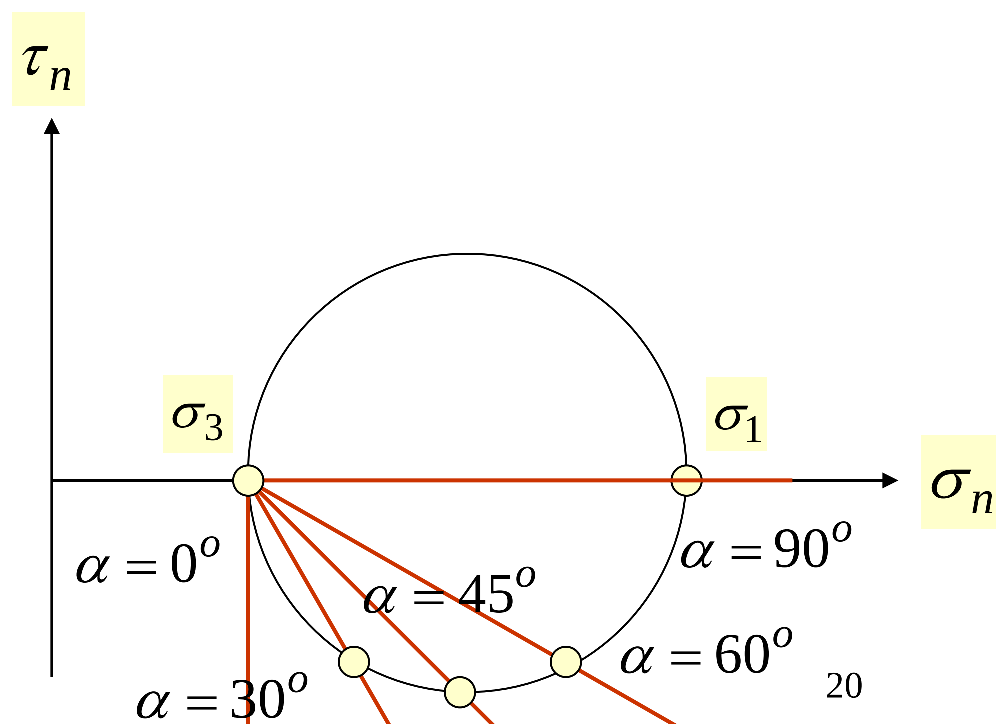
$$(x - x_o)^2 + y^2 = R^2$$

Mohr's circle

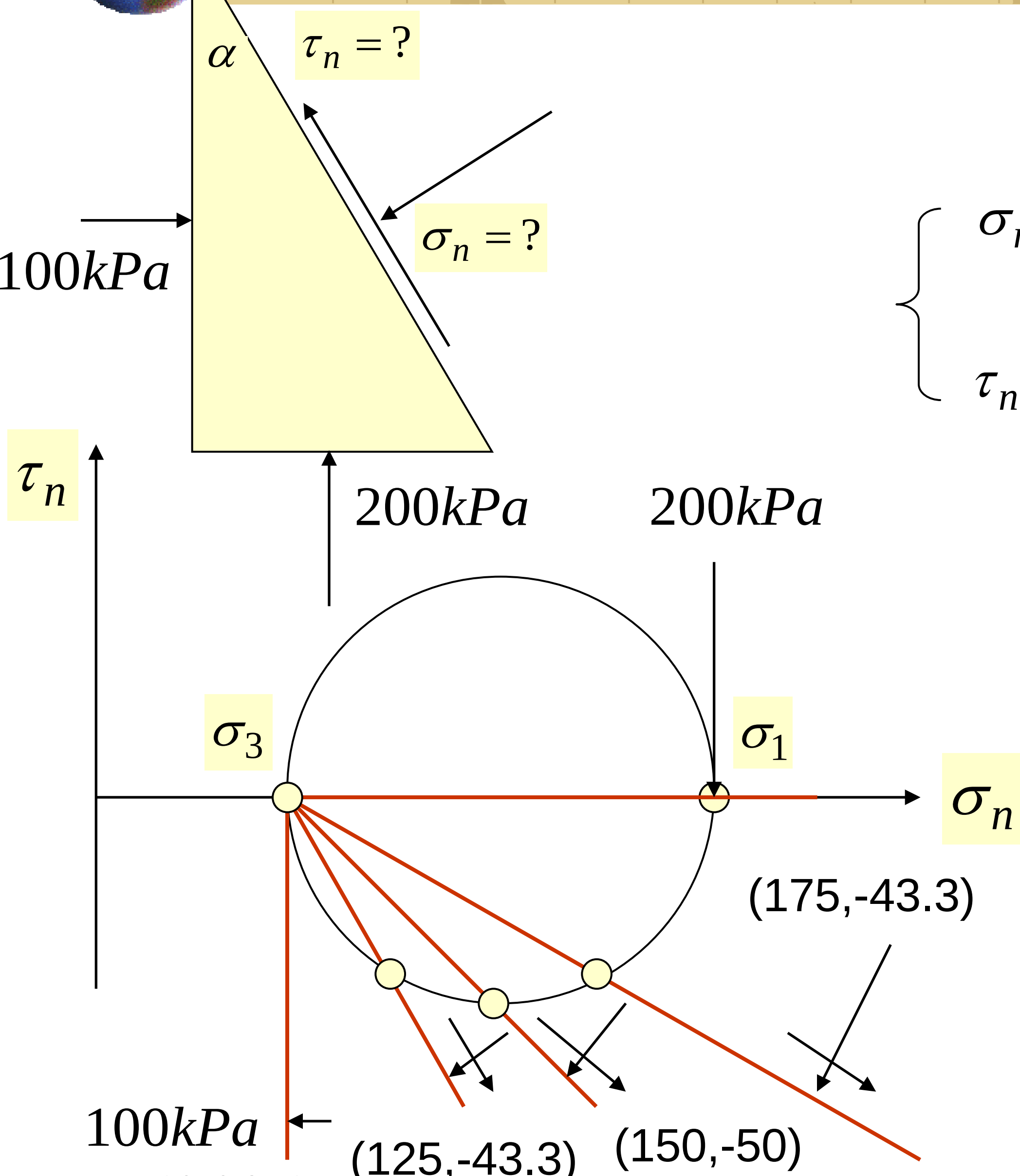
$$(x - x_o)^2 + y^2 = R^2$$

$$\begin{cases} \sigma_n = \frac{1}{2}(\sigma_1 + \sigma_3) - \frac{1}{2}(\sigma_1 - \sigma_3) \times \cos 2\alpha \\ \tau_n = -\frac{1}{2}(\sigma_1 - \sigma_3) \times \sin 2\alpha \end{cases}$$





Mohr's circle



$$(x - x_o)^2 + y^2 = R^2$$

$$\begin{cases} \sigma_n = \frac{1}{2}(\sigma_1 + \sigma_3) - \frac{1}{2}(\sigma_1 - \sigma_3) \times \cos 2\alpha \\ \tau_n = \qquad -\frac{1}{2}(\sigma_1 - \sigma_3) \times \sin 2\alpha \end{cases}$$

$$\alpha = 0^o, (150 - 50 \times 1, -50 \times 0)$$

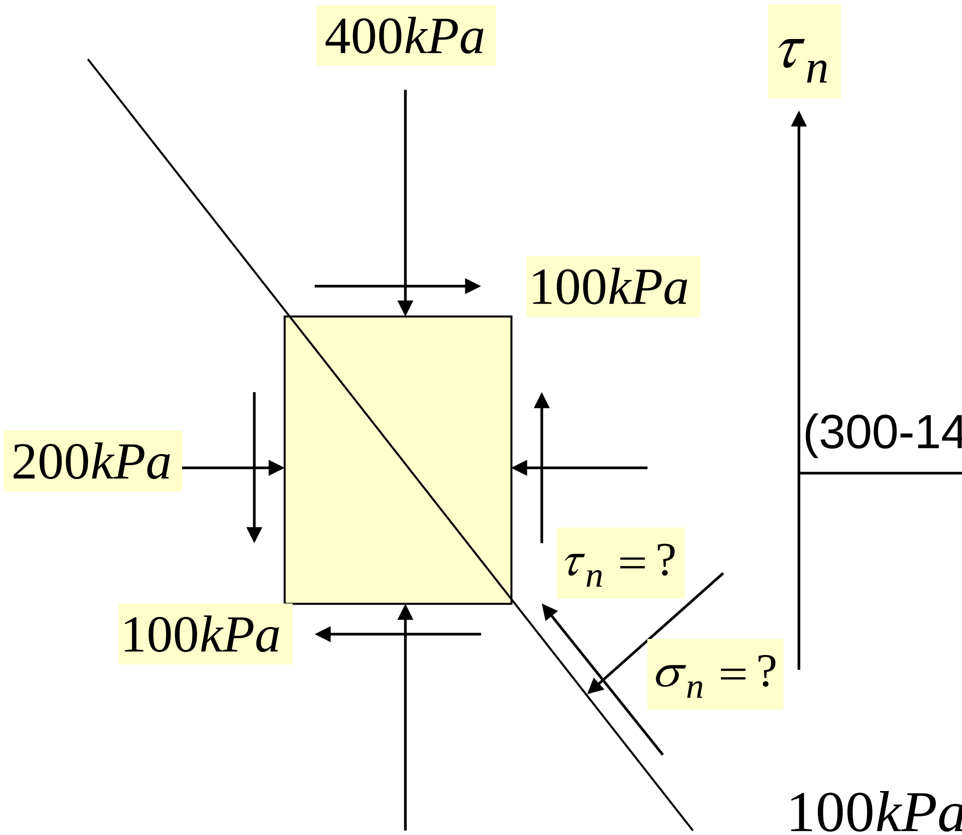
$$\alpha = 30^o, (150 - 50 \times \frac{1}{2}, -50 \times \frac{\sqrt{3}}{2})$$

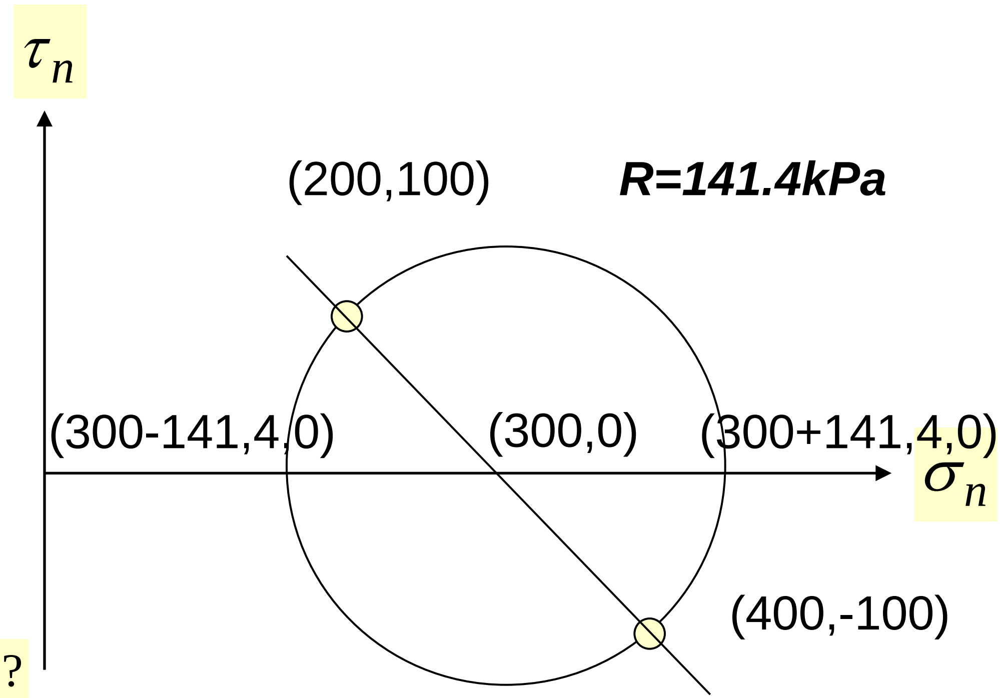
$$\alpha = 45^o, (150 - 50 \times 0, -50 \times 1)$$

$$\alpha = 60^o, (150 + 50 \times \frac{1}{2}, -50 \times \frac{\sqrt{3}}{2})$$

$$\alpha = 90^o, (150 + 50 \times 1, -50 \times 0)$$

Construct a Mohr circle

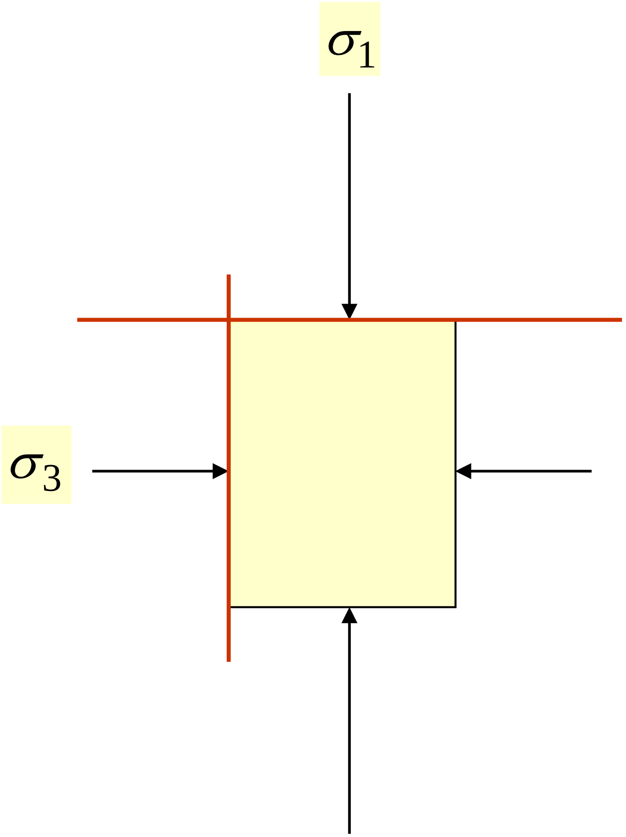


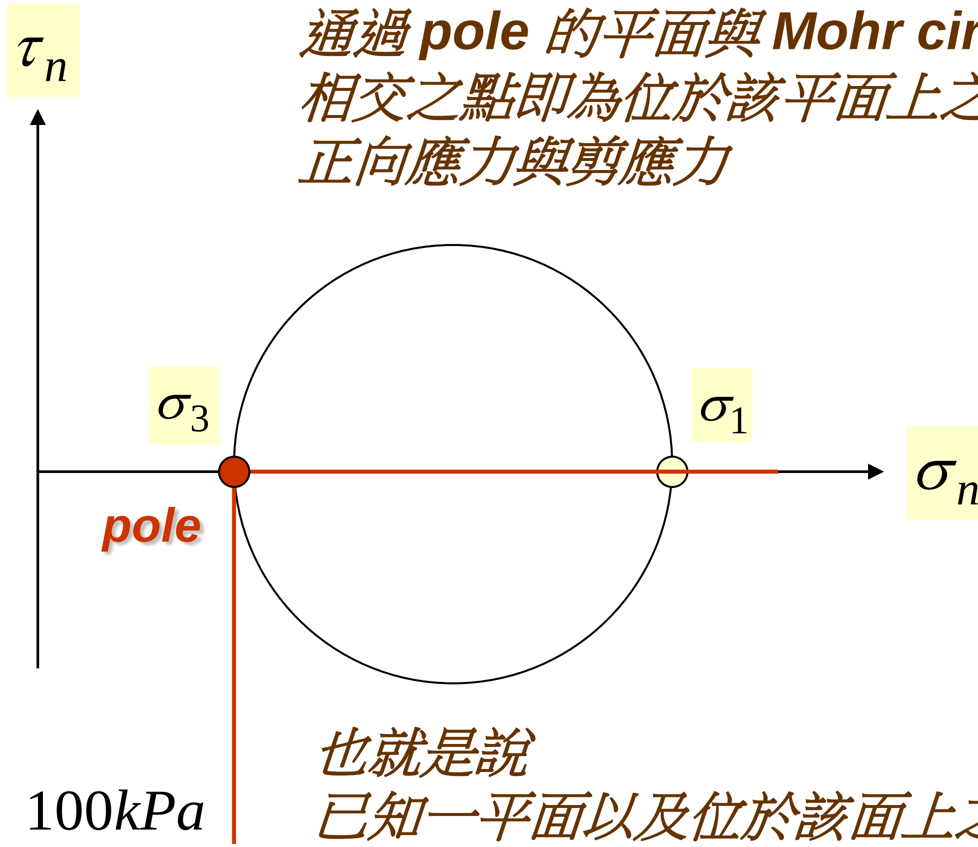


圓心？
半徑？
最大主應力？
最小主應力？

主應力方向？

Pole method

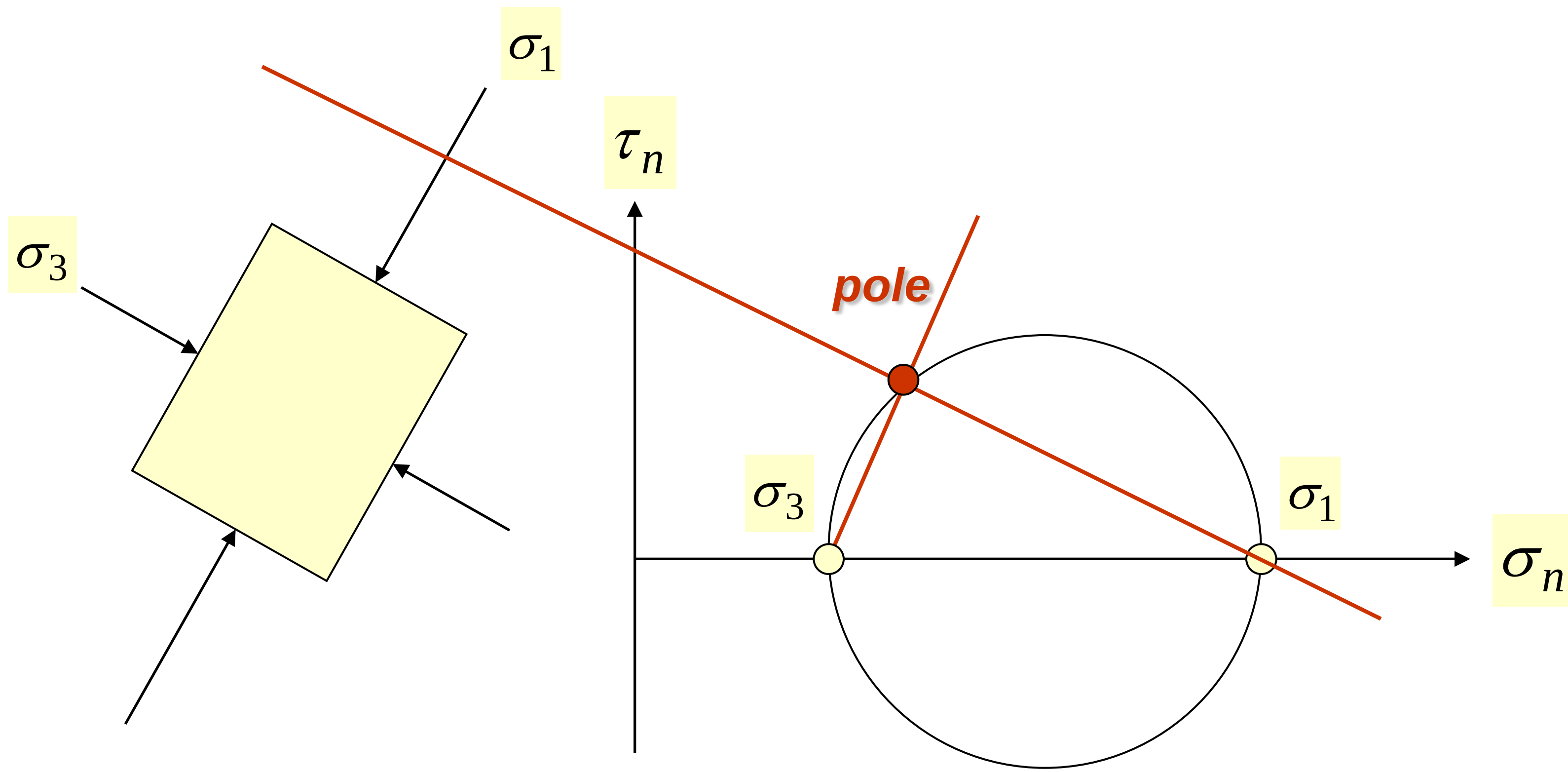




通過 pole 的平面與 Mohr circle 相交之點即為位於該平面上之正向應力與剪應力

也就是說
已知一平面以及位於該面上之正向應力與剪應力
通過該點與 Mohr circle 相交點即為 pole

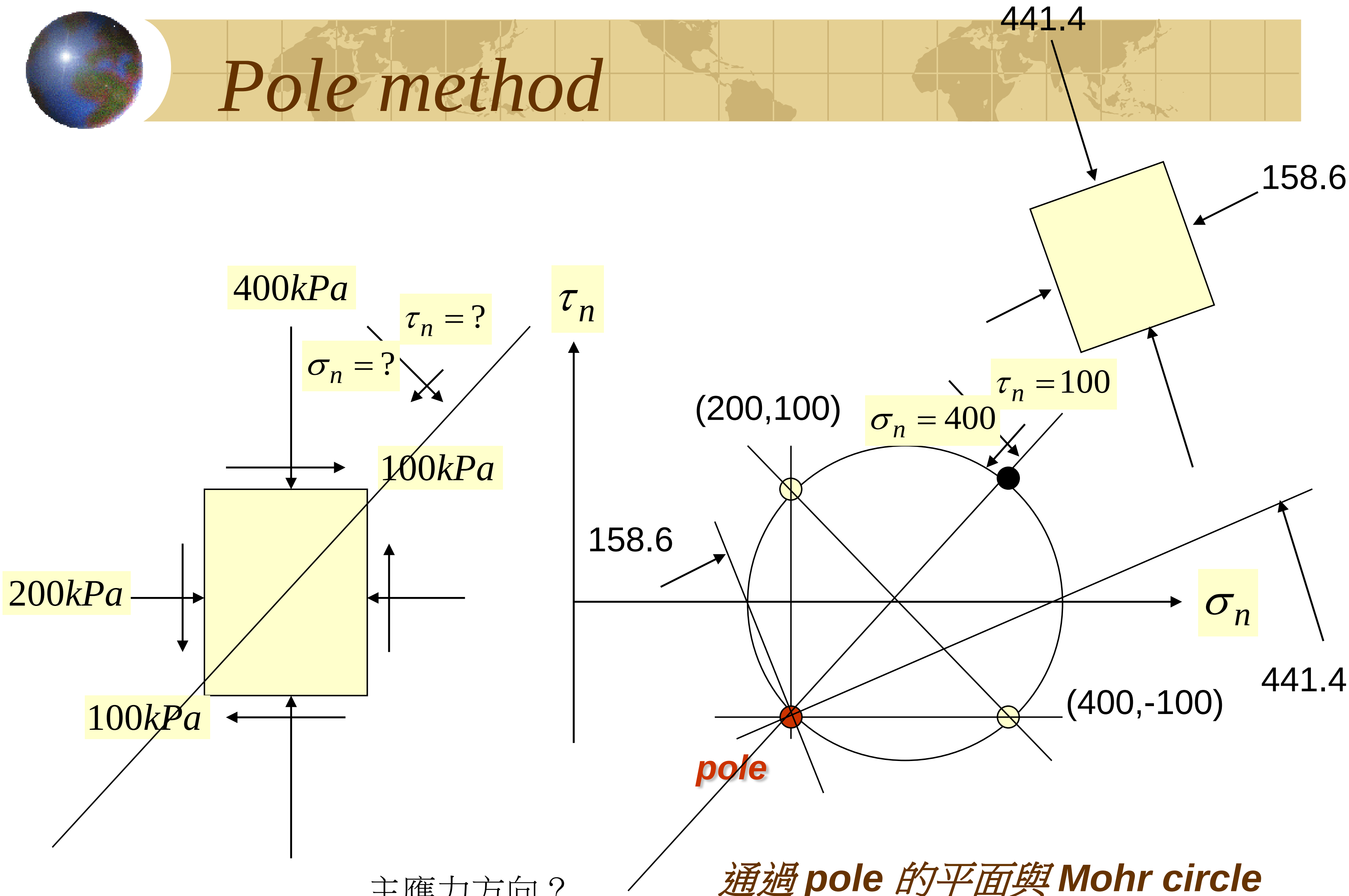
Pole method



$100kPa$

也就是說
已知一平面以及位於該面上之
正向應力與剪應力
通過該點與 **Mohr circle**
相交點即為 **pole**

Pole method



主應力方向？

通過 pole 的平面與 Mohr circle 相交之點即為位於該平面上之正向應力與剪應力

▶ **Example 8.4**

Given. Figure E8.4-1.

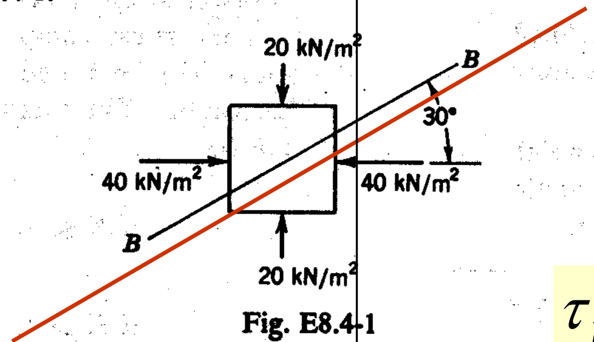


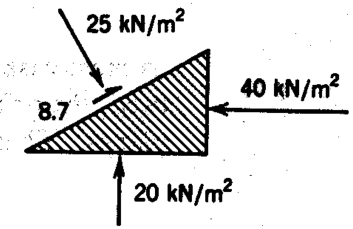
Fig. E8.4-1

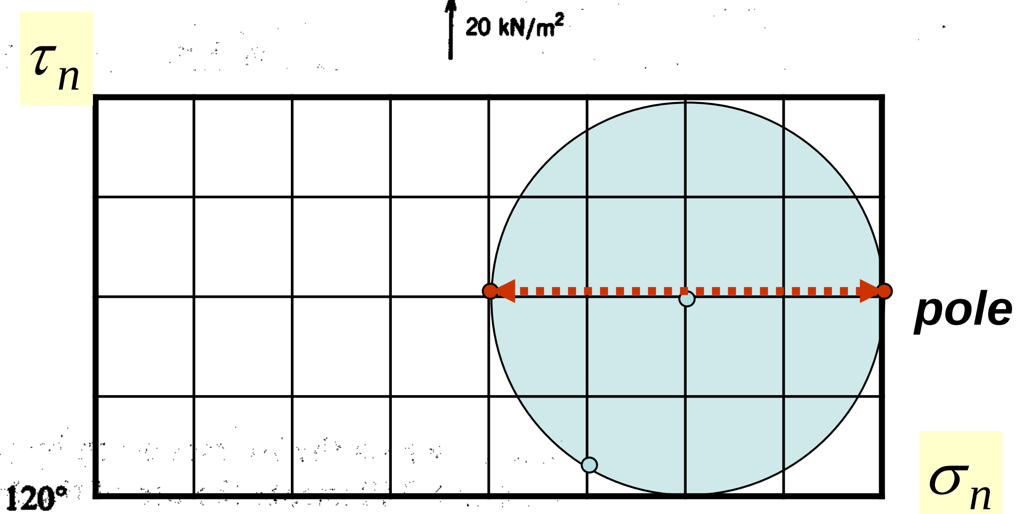
Find. Stresses on plane *B–B*.
Solution. Use Fig. E8.4-2.

1. Locate points with co-ordinates (40, 0) and (20, 0).
2. Draw circle, using these points to define diameter.
3. Draw line $A'A'$ through point (20, 0) and parallel to plane on which stress (20, 0) acts.
4. Intersection of $A'A'$ with Mohr circle at point (40, 0) is the origin of planes.
5. Draw line $B'B'$ through O_P parallel to BB.
6. Read coordinates of point X where $B'B'$ intersects Mohr circle.

Answer. See Fig. E8.4-3.

$$\text{on } BB \quad \begin{cases} \sigma = 25 \text{ kN/m}^2 \\ \tau = -8.7 \text{ kN/m}^2 \end{cases}$$





Solution Using Eqs. 8.6 and 8.7.

$$\sigma_1 = 40 \text{ kN/m}^2 \quad \sigma_3 = 20 \text{ kN/m}^2 \quad \theta = 120°$$

$$\sigma_\theta = \frac{40+20}{2} + \frac{40-20}{2}\cos 240° = 30 - 10\cos 60° = 25 \text{ kN/m}^2$$

$$\tau_\theta = \frac{40-20}{2}\sin 240° = -10\sin 60° = -8.66 \text{ kN/m}^2$$ ◀

(*Questions for student.* Why is $\theta = 120°$? Would result be different if $\theta = 300°$?)

Given. Figure E8.5-1.

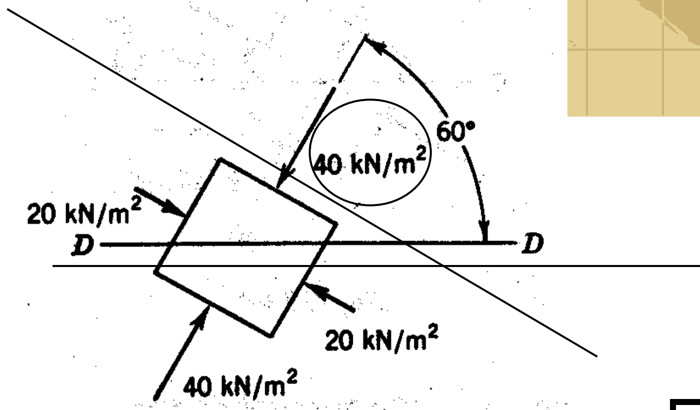
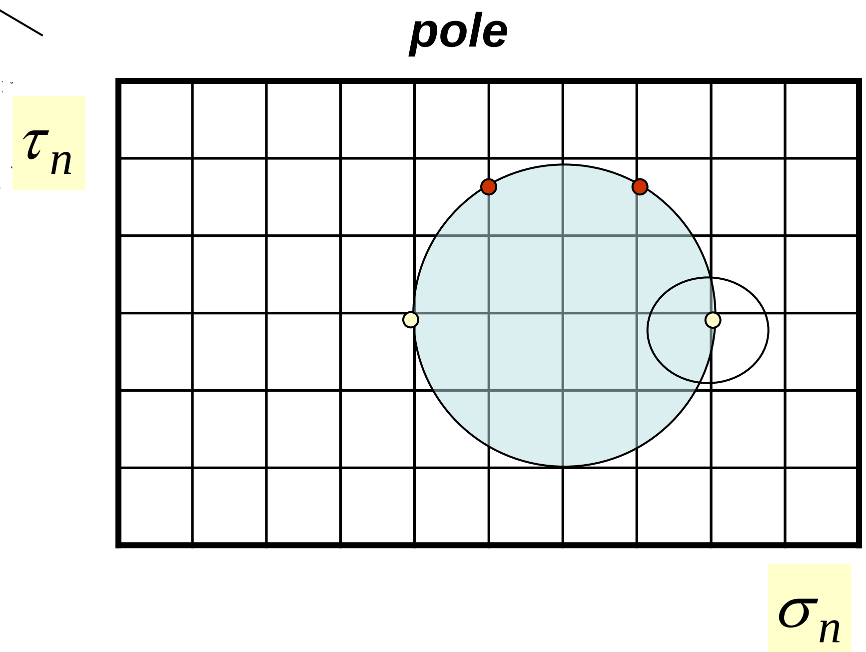
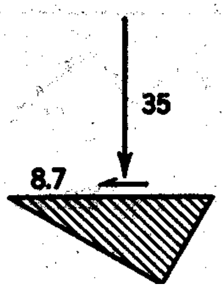


Fig. E8.5-1

Find. Stresses on horizontal plane DD.



on DD $\begin{cases} \sigma = 35 \text{ kN/m}^2 \\ \tau = 8.7 \text{ kN/m}^2 \end{cases}$



2. Draw Mohr circle.
3. Draw line $A'A'$ through (20, 0) parallel to plane upon which stress (20, 0) acts.
4. Intersection of $A'A'$ with Mohr circle gives O_P.
5. Draw line $D'D'$ parallel to plane DD.
6. Intersection X gives desired stresses